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GAME-THEORETIC ANTENNA SUBSET SELECTION FOR MULTI-USER MIMO SYSTEMS

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Universidade Federal do Ceará
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Game-Theoretic Antenna Subset Selection for Multi-User MIMO Systems

Master of Science Thesis

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Dissertação submetida à Coordenação do Programa de Pós-graduação em Engenharia de Teleinformática, da Universidade Federal do Ceará, como parte dos requisitos para obtenção do grau de Mestre em Engenharia de Teleinformática.

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To my family.

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Resumo

Nesta dissertação de mestrado, um novo algoritmo de seleção de antenas é proposto usando uma abordagem baseada na teoria dos jogos. Assume-se que cada estação base em um grupo de estações base é vinculada a um terminal associado como um par receptor-transmissor. Esses pares receptores-transmissores reutilizam recursos de canal, tal que cada terminal móvel representa uma fonte de interferência externa –também referenciada como interferência multi-usuário ou (MUI) –para outros terminais móveis em células vizinhas que estão reutilizando todos ou parte dos mesmos recursos de canal. Consequentemente, as estações base implementam um algoritmo baseado em jogo de forma a suavizar a MUI para os sinais múltiplas-entradas-múltiplas-saídas (MIMO) em uplink recebidos de seus terminais móveis associados. Particularmente, cada estação base funciona como um jogador em um jogo, no qual a ação de jogo permitida é a seleção da matriz de pré-codificação a ser usada para transmissão MIMO de uplink para a estação base de um terminal móvel associado. Resultados de simulação mostram que o algoritmo proposto tem um bom desempenho em termos de probabilidade de erro média consistindo-se em um conceito de solução baseado em pontos de equilíbrio de Nash.

Palavras-chave: Suavização de Interferência, MIMO, Teoria dos Jogos, Algoritmos Distribuídos.

Abstract

In this Master's thesis, a novel antenna subset selection algorithm is proposed using a game-theoretical approach. It is assumed that each base station in a group of base stations is linked to an associated terminal as a receiver-transmitter pair. These receiver-transmitter pairs reuse channel resources, such that each mobile terminal represents a source of other-cell interference (also referred to as multi-user interference or MUI) for other mobile terminals in neighboring cells that are reusing all or some of the same channel resources. Accordingly, the base stations implement a gaming-based algorithm to mitigate MUI for the multiple-input-multiple-output (MIMO) uplink signals received from their associated mobile terminals. More particularly, each base station functions as a player in a game, in which the allowed gaming action is the selection of the precoding matrix to be used for MIMO uplink transmission to the base station from an associated mobile terminal. Simulation results show that the proposed algorithm has a good perfomance in terms of average error probability consisting of a solution concept based on Nash equilibrium points.

Key-words: Interference Mitigation, MIMO, Game Theory, Distributed Algorithms.

List of Acronyms

BER	bit error rate
BPSK	binary phase shift keying
BSC	base-station controller
BS	base-station
CNPq	National Council for Scientific and Technological Development
CSI	channel state information
GRASS	Game-theoRetic Antenna Subset Selection
GTEL	Wireless Telecommunications Research Group
i.i.d	independent identically distributed
INR	interference-to-noise ratio
IWF	iterative waterfilling
MIMO	multiple-input-multiple-output
MISO	multiple-input-single-output
MMSE	minimum mean-square error
MMSV	maximum minimum singular value
MSV	minimum singular value
MUI	multi-user interference
NE	Nash equilibrium
RF	radio frequency
SC-FDMA	single-carrier frequency-division multiple access
SINR	signal-to-interference-plus-noise ratio
SIR	signal-to-interference ratio
SM	spatial multiplexing
SNR	signal-to-noise ratio

STBC space-time block code

SVD singular value decomposition

UE user equipment

ZMCSCG zero-mean circularly symmetric complex Gaussian

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Chapter 1

Introduction

1.1 Context of the Problem

In a scenario where higher and higher spectral efficiency is necessary, sharing of resources is mandatory. This issue requires flexible networks and also yields a competitive nature in modern communication systems.

Multiple transmit and receive antennas (for multiple-input-multiple-output (MIMO) transmit/receive processing) can be used to mitigate multi-user interference (MUI) if they include some intelligent transmission technique. For instance, the use of directional antennas and antenna arrays has long been recognized as an effective technique to reduce MUI [1]. If multiple antennas are also employed to perform spatial multiplexing (SM), where data are transmitted over multiple transmit antennas [2], the spectral efficiency can be further increased.

By using only a subset of the available transmit antennas, it is possible to mitigate the MUI by using the excess antennas to obtain a diversity gain. With a simple linear precoding process [3], the antenna subset which yields the least MUI for each user is selected. After that, SM is performed in the selected antennas.

Different criteria have been used for the subset selection such as maximizing the channel capacity [4], maximizing the post-processing signal-to-noise ratio (SNR) [5] and maximizing the minimum singular value (MSV) of the channel matrix [5]. Those criteria can be employed straightforwardly in scenarios with MUI. For instance, the post-processing SNR maximization criterion becomes post-processing signal-to-interference-plus-noise ratio (SINR) maximization. Also, it is possible to perform the antenna subset selection through centralized optimization by exhaustive search over all possible antenna combinations.

1.2 Motivation and Objectives

A potential drawback of MIMO systems is the increase in the overall system cost. An antenna element and signal processing become cheaper but a radio frequency (RF) element do not. Thus, antenna selection algorithms emerge as a good solution for decreasing the overall system cost with some (or no) performance loss [6]. Moreover, antenna selection schemes provide good diversity gain and the diversity order obtained is proportional to the number of antenna elements, not to the number of RF elements [6].

The information feedback channel is considered limited in terms of bit rate. Thus, the exhaustive search approach might be not feasible in practical systems due to the high computational complexity and excessive signaling load requirements to obtain the optimal solution. Moreover, linear receivers are widely used to separate the incoming data streams mainly due to their simplicity. But, the capacity maximization criterion is not specialized to this kind of receivers and it probably might result in a suboptimal solution. The maximum minimum singular value (MMSV) criterion does not take into account the influence of MUI. That is, it does not work well in a regime of low signal-to-interference ratio (SIR).

Game theory has also been adopted to solve many problems in communication systems by modeling such systems in a distributed way [7–10]. In particular, game theory has been employed to determine optimal precoding/multiplexing matrices for multipoint-to-multipoint systems [11]. However, it does not appear that any existing work applies game theory to the problem of antenna subset selection in uplink multi-user communications, via a linear precoding process.

Objectives

Concisely, this Master's thesis proposes an algorithm whose objectives are:

- applying a linear precoding technique using a finite set of precoders;
- making use of distributed computation based on a game-theoretic approach;
- exploring antenna subset selection in an interfering environment;
- significantly mitigating the inherent MUI in multi-cell scenarios.

1.3 Scientific Production and Contributions

Throughout the Master's course, a United States patent regular application related to the distributed algorithm proposed herein has been filed, with the following information:

- P29771-US2 "*A Method and Apparatus for Reducing Multi-User-Interference in a Wireless Communication System*", **I. M. Guerreiro**, D. C. Moreira, I. L. J. da Silva, C. C. Cavalcante, and D. Hui, C&B Ref. 4015-6670.

It is worth noting that the proposed algorithm had not been disclosed before late September 2009 due to the patent processing. Accordingly, further scientific productions, e.g., journal papers and conference papers, are in the process of being written and submitted only now.

The kernel of this Master's thesis has been developed in the context of two research projects of a research cooperation between GTEL and ERICSSON Research. Thus, during the Master's research three technical reports have been produced and one is in the process of being written. The list follows below:

- "*Game-Theoretic Antenna Subset Selection in MIMO Multiuser Systems (GRASS)*", **I. M. Guerreiro**, Third Intermediate Technical Report (TR03) of UFC.23 Project, To be delivered in February 2010.
- "*Game-Theoretic Antenna Subset Selection in MIMO Multiuser Systems (GRASS)*", **I. M. Guerreiro**, Second Intermediate Technical Report (TR02) of UFC.23 Project, August 2009.
- "*Game-Theoretic Antenna Subset Selection in MIMO Multiuser Systems*", **I. M. Guerreiro** and D. C. Moreira, First Intermediate Technical Report (TR01) of UFC.23 Project, February 2009.
- "*Simulation Tool: Link Level Cross-layer Simulator (LLCROSS)*", D. C. Moreira, C. A. Araújo and **I. M. Guerreiro**, Final Technical Report (TR04) of UFC.19 Project, July 2008.

As for contributions, this Master's thesis presents an algorithm which applies to wireless communication networks, enabling a reduction in multi-user interference (MUI) in wireless communication networks which employ multiple-input-multiple-output (MIMO) transmission. By introducing the concept of reaching an equilibrium point in the system, it may optimize the exchange of information performed in real communication systems.

1.4 Document Organization

The following notation is used throughout this document. Uppercase and lowercase boldface denote matrices and vectors, respectively. The operators $\mathbb{E}\{\ \cdot\ \}$, $\|\cdot\|$, $\mathrm{D}[\ \cdot\ ]$, $|\cdot|$, $\lceil\ \cdot\ \rceil$, $(\cdot)^H$ and $\mathrm{tr}(\cdot)$ stand for expectation, norm operator, decision operator, modulus, ceil, hermitian and trace operator, respectively.

The remainder of this document is organized as follows.

Chapter 2 – Basic concepts of game-theory, e.g., fundamental components and equilibrium notion, and the system model are presented in order to allow a smooth understanding of this Master's thesis.

Chapter 3 – The game structure is described, presenting all the mathematical basis used herein. Besides, the proposed algorithm (Game-theoRetic Antenna Subset Selection (GRASS)) is described, showing its key features, time diagrams, flow charts, as well as the associated information exchange.

Chapter 4 – A numerical analysis is provided for the simulation results obtained via Monte Carlo simulations.

Chapter 5 The overall work presented herein is concluded analyzing the aims achieved and pointing out the perspectives for future work.

Chapter 2

Fundamentals: Game Theory and Interference in Wireless Systems

2.1 Brief History of Game Theory

Game theory is a branch of applied mathematics which provides a mathematical basis for the analysis of interactive decision-making processes [12]. It also provides tools for predicting what might happen when individuals with conflicting interests interact, or more generally, for analyzing optimization problems with multiple conflicting objective functions. It uses models to study interactions with formalized incentive structures called “games”. Such models are based on mathematical models of conflict and cooperation among intelligent and rational decision makers. study An individual is said to be “rational” if each one of his[1] decision-making behaviors is consistent with the maximization of expected utility. Furthermore, an individual is also said to be “inteligent” if this individual understands everything about the structure of the situation, including the fact that others are intelligent, rational decision makers. In fact, these two assumptions are fundamental for the structure of the game.

Game theory can be applied in a variety of fields, including economics, international relations, evolutionary biology, political science, and military strategy. Game theorists study the predicted and actual behaviors, as well as optimal strategies, of individuals in games.

The history of game theory originates from the works [13] by Waldegrave (1713), Cournot (1838), Darwin (1871), Edgeworth (1881), Zermelo (1913), Borel (1921), and Ville (1938). In modern times the major works by Von Neumann and Morgenstern, e.g., *Theory of Games and Economic Behavior* published in 1944, provided an axiomatic development of utility theory, which dominates the current economic thought and also introduced the formal notion of a cooperative game. Another important name in the history of game theory is Jonh Nash, who contributed to the development of both non-cooperative and cooperative game theory [14–16]. In [14], Nash contributed (probably this is his most important contribution) with the existence proof of an equilibrium in finite[2] non-cooperative games, the so-called Nash Equilibrium.

[1]Without any preference to sexes, both an individual and a decision maker, in this document, are always referred to as a “he”. But it could equally well be a “she”.

[2]The class of games in which the players have a *finite* number of alternatives to choose from is called finite games.

2.2 Non-cooperative Static Games

In a non-cooperative game, each player, belonging to a set of players, adjusts his strategy to optimize his own ability (utility) to compete with others. It is relatively easy to delineate the main ingredients of a conflict situation: a player has to make a decision and each possible decision leads to a different outcome or result, which are valued differently by that player. This player may not be the only one who decides about a particular outcome; a series of decisions by several individuals may be necessary. If all these players value the possible outcomes differently, the seeds for a conflict situation are there. The players involved do not always have complete control over the outcome. Sometimes there are uncertainties which influence the outcome in an unpredictable way. Under such circumstances, the outcome is (partly) based on data not yet known and not determined by the other players' decisions.

Strategy

A strategy is a complete contingent plan, or a decision rule, that defines the action a player will select in every different state of the game. It is worth distinguishing between actions and strategies. A simple real-world situation can distinguish them: if a player has to decide about what to do the next day, and the options are fishing and going to work, then a strategy is "if the weather report predicts dry weather, then that player will go fishing, otherwise he will go to his office". What actually will be done depends on quantities not yet known and not controlled by the decision maker, e.g., the wheather condition. On the other hand, any consequence of such a strategy, after the unknown quantities are realized, is called an action. By the way, a constant strategy coincides with the notion of action. A player has *pure strategies* when he always picks a single strategy in his strategy set[3]. An alternative is for a player to randomize over his strategy set. In this case, that player has *mixed strategies*. In other words, a mixed strategy is an assignment of a probability to each pure strategy, whereas a pure strategy is selected with probability 1 and every other strategy with probability 0. This work will only deal with pure strategies, but further definitions can be found in [7, 12].

Utility

A utility (payoff) function represents the motivations of players. A utility function for a given player assigns a (real) number for every possible outcome of the game with the property that a higher (or lower) number implies that the outcome is more preferred. Therefore, a player's strategy may be formulated as "maximizing (minimize) his utility (cost)".

Strategic Form Game

Strategic form (or normal form) is a matrix representation of a simultaneous game. For a two-player game, one player is the "row" (two-dimensional matrix) and the other is the "column". For Q players, each one is a dimension of the Q-dimensional matrix. Each dimension (e.g., row or column for two-dimensional matrix) represents a strategy, and each entry represents the utility to each player for every combination of strategies.

Nash Equilibrium

The Nash equilibrium (NE) is the most common solution concept of a game. It is a joint strategy where no player can increase his utility function by unilaterally deviating [7]. That is, no player has anything to gain by changing his strategy while the other players keep their unchanged. Another interpretation of Nash equilibrium is that it is a mutual best response from each player to other player's strategies. It is worth mentioning that a NE is not always

[3] A player's strategy set defines what strategies are available for that player to play.

clearly efficient (or Pareto optimal)[4]. Nevertheless, the NE remains the fundamental concept in game theory. In [17], Nash proved that every finite strategic-form game has at least one mixed-strategy NE. As for pure-strategy NE, the uniqueness or even existence of such a NE is not guaranteed. For it, some desirable properties of the structure of a game must be established. However, this is beyond the scope of this Master's thesis.

Further Game Aspects

A game can be classified according to multiple aspects. Three of them are adopted in this Master's thesis so that the other ones are beyond the scope of this work. Further definitions can be found in [7, 12, 18].

i. Nonzero-Sum Games: The conflicts in a game determine if a given game is classified as either zero-sum or nonzero-sum. There is competition in both cases, but in a zero-sum game a gain for a player is exactly a loss for the other. That is, the summation of the players' utility equals zero. Otherwise, the game is said to be a nonzero-sum one.

ii. Static Games: A static game is defined to as one in which all players make decisions (or select a strategy) simultaneously, without knowledge of the other players' strategies. Although the decisions may be made at different points in time, the game is simultaneous because each player has no information about the decision of others.

iii. Non-cooperative Games: As for cooperation in a game, a game is said to be non-coopertive when all players make decisions independently. Thus, while they may be able to cooperate, any cooperation must be self-enforcing. Furthermore, players can communicate with each other but cannot make a deal.

A classical (and fundamental) example game is the well-known "Prisoner's Dilemma" [18], which was popularized by the mathematician Albert W. Tucker. This example shows a hypothetical situation: Two criminals, e.g., Bonnie and Clyde, are arrested for commiting a crime in unison, but the police do not have enough proof to convict either. Thus, the police separate the two and offer a deal: If Bonnie testifies to convict (betray) Clyde , she will get a sentence of 10 years if he also betrays her, or go free otherwise. However, if Bonnie does not betray (i.e., be silent) Clyde, she will get a maximum sentence of 20 years if Clyde betrays her, or get a reduced sentence of 2 years. The same deal is offered to Clyde. The strategic form is shown in the matrix below.

	Clyde		
Bonnie	10,10	0,20	Betray
	20, 0	2, 2	Not Betray
	Betray	Not Betray	

Thus, each player's strategy is "Betray" or "Not Betray". This game is classified as non-cooperative and static because the players do not exchange any information with each other. Besides, they make decisions independently (i.e., separately in different rooms) and simultaneously. The only equilibrium in this example game is "Betray\Betray". However, this solution is inefficient because "Not Betray\Not Betray" provides a better output than the NE. This Pareto-optimal solution can be reached if Bonnie and Clyde cooperate.

[4]A Pareto-optimal solution is a joint decision of the players made in cooperation such that no other solution can improve the performance of at least one them, without degrading the performance of the other.

As another example, in wireless communications, to obtain information such as channel conditions, signaling is performed so that resource allocation (or other type of optimization problem) can result in an optimal solution. However, signaling in current wireless networks has considerable overhead for communications. Thus, reducing the overhead can greatly increase the spectrum utilization, increase the number of users, and improve the network performance. One of the possible ways to reduce overhead is to do resource optimization by using only local information. This is especially important if the system topology is distributed. In some wireless network scenarios, it is hard for an individual user to know the channel conditions of the other users. The users cannot cooperate with each other. They act selfishly to maximize their own performance in a distributed fashion. Such a fact motivate us to adopt game theory.

With the above understanding in mind, this Master's thesis proposes an antenna subset selection game, presenting a distributed algorithm named GRASS, for a competitive MIMO system in an uplink multi-user scenario. The game structure aims at maximizing the minimum SINR per stream of each mobile terminal.

To make that selection competitive among the base-stations (BSs) "playing the game," each round of game play involves each BS making its own precoding matrix selection while assuming that the other BSs hold their selections fixed. For example, during each round of game play a BS determines a covariance estimate for MUI that depends on the precoding matrices in use at the other mobile terminals, and it evaluates a utility function over the range of available precoding matrix selections. That utility function depends on the covariance estimate and on the particular selection of the precoding matrix for the associated mobile terminal. As an example, the utility function maximizes the minimum SINR determined for the MIMO uplink signals from the associated mobile terminal, over all the MIMO streams. Once the quality-maximizing precoding matrix is found and selected, it can be sent to the associated mobile terminal (e.g., by identifying its index within a predefined set of precoding matrices).

As such, in each round of game play, each BS picks the precoding matrix that maximizes received uplink quality at the BS, for the BS's associated mobile terminal, while assuming that the other BSs are holding the precoding matrices of their associated mobile terminal fixed. However, after each round of game play, the updated precoding matrix selection can be exchanged among all BSs, or estimated/inferred by each BS and a new round of game play is commenced according to the new precoding matrix selections.

Game play can be iterated in this fashion until an equilibrium point is reached by the BS as regards precoding matrix selections, or until an allowed iteration limit is reached—to guard against non-convergence problems. If the iteration limit bound is reached, each given BS uses another algorithm—e.g., a non-iterative algorithm—to select the precoding matrix to be used by its associated mobile terminal. For example, the BS may use a MMSV algorithm for precoding matrix selection.

2.3 System Model

Consider a multiuser scenario with K users spread over Q cells. The reuse factor is equal to one and there is no intracell interference. On the other hand, there are co-channel transmit-receiver pairs (links) in uplink communication that share time and bandwidth resources causing intercell interference. The scenario described above can be implemented by adopting a multiple access technique in each cell, such as single-carrier frequency-division multiple access (SC-FDMA) [19]. Therefore, for a given set of resources, there are at most Q

neighboring links, each user equipment (UE) in a cell, that interfere with each other. Thus, considering the worst case, the set of neighboring links is defined as follows:

$$\Gamma = \{1, \ldots, Q\}. \tag{2.1}$$

In addition, each BS may be connected to a base-station controller (BSC) through, for example, a high-speed wired link in order to exchange information, if this feature is needed. Fig. 2.1 illustrates a 2-cell scenario where two items of UE share resources (the remaining $K-2$ users are omitted). For convenience, each item of UE is simply referred to as a UE.

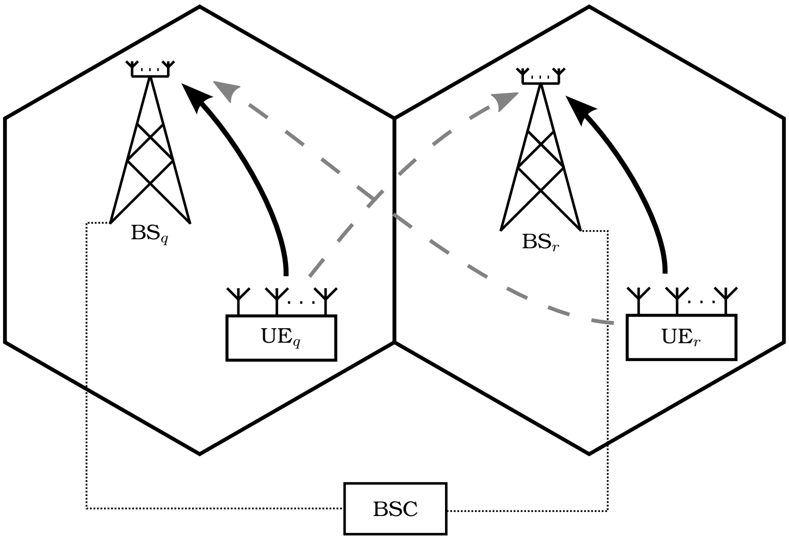

Figure 2.1: Example of a general 2-cell scenario.

More particularly, Fig. 2.1 depicts an example wireless communication network, including two cells, each including a corresponding BS. The two BSs are connected to a (centralized) BSC. The arrangement provides cell-based wireless communication service to two UEs. Of interest herein is the case where one BS (e.g., BS_q) supports a given UE (e.g., UE_q) in a first cell, and another neighboring BS (e.g., BS_r) supports another UE (e.g., UE_r) on some or all of the same channel resources.

In this scenario, UE_q acts as a source of interference bearing on reception of uplink transmissions between UE_r and BS_r. Likewise, UE_r acts as a source of interference bearing on reception of uplink transmissions between UE_q and BS_q (multi-user interference or MUI). If BS_q and BS_r "play" an interference reduction game between them, iterative game play can drive each BS to reduce the MUI.

Definition 2.1—configuration 1:

The configuration 1 considers that there exists a BSC controling error-free information exchange among BSs. In addition to this, the link from a BS to the BSC is called *direct wired link* and the opposite is called *reverse wired link*. Besides, the downlink (link from each BS to its UE) is limited in terms of bit rate and it is called *limited-feedback link*, as can be seen in Fig 2.2.

Definition 2.2—configuration 2:

In configuration 2, BSs are not connected to a BSC. Information exchange is performed through a pilot-aided process (e.g., pilot-aided channel estimation regarding estimation errors). That is, each neighboring BS sends a pilot signal and BSs estimate the information needed to play the game. As in configuration 1, the downlink (link from

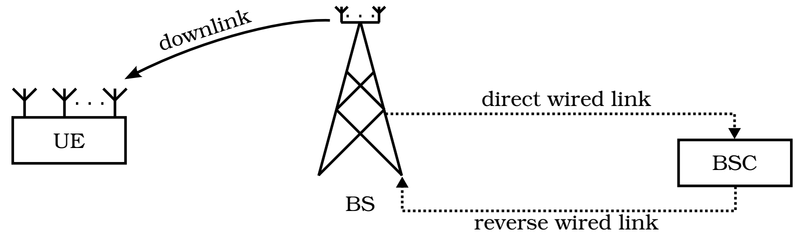

Figure 2.2: Link types in configuration 1.

each BS to its UE) is limited in terms of bit rate and it is called *limited-feedback link*. Fig 2.3 illustrates a general scenario with configuration 2.

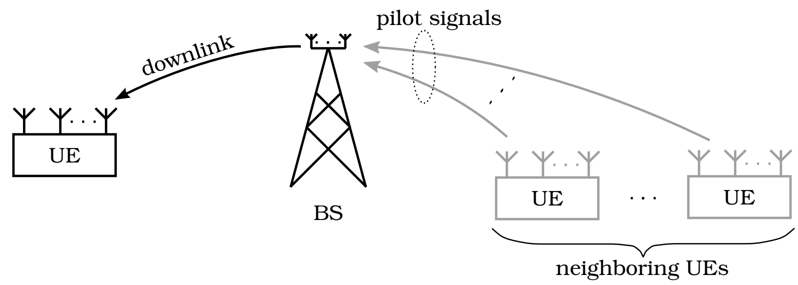

Figure 2.3: Link types in configuration 2.

In more details, the UE_q is the q-th source that transmits precoded and spatially multiplexed symbol vectors $\mathbf{x}_q$ to the q-th BS (BS_q). The symbol vectors $\mathbf{x}_q$ are defined as

$$\mathbf{x}_q = \sqrt{\frac{1}{N}}\mathbf{F}_q\mathbf{s}_q, \quad \forall q \in \Gamma, \tag{2.2}$$

where $\mathbf{F}_q$ is the $M_T \times N$ precoding matrix and $\mathbf{s}_q$ is the $N \times 1$ vector of SM symbols s_k defined as

$$\mathbf{s}_q = [s_k]_{k \in \mathcal{N} \triangleq \{1,\ldots,N\}}.$$

One may assume M_T, N and M_R as being the number of available transmit antennas, the number of RF chains and the number of receive antennas, respectively. The q-th base station, BS_q, as the q-th destination also receives interfering signals from the other $Q-1$ links. Furthermore, $\mathcal{N}$ denotes the index set of the uncoded symbol streams.

The sampled symbol vector received by the q-th BS is

$$\mathbf{y}_q = \mathbf{H}_{qq}\mathbf{x}_q + \sum_{r=1, r\neq q}^{Q} \sqrt{g_{rq}}\mathbf{H}_{rq}\mathbf{x}_r + \mathbf{n}_q, \tag{2.3}$$

where $\mathbf{H}_{qq}$ is the channel matrix between source q and destination q and $\mathbf{n}_q$ is the zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise vector with covariance matrix $N_o\mathbf{I}$. On the right-hand side of Eq. (2.3), the second term refers to the MUI caused by the other links and received by the q-th BS. The fading between each transmit and receive antenna is assumed to be independent, modeled by ZMCSCG random variables and *quasi*-static over a data block of L SM symbols. Also, it is assumed that each BS knows the channel state information (CSI) for its associated UE perfectly. Further, each BS may know the CSI for the other, interfering UEs. The constant g_{rq} is a gain that depends on the path loss of each interfering signal, here modeled in a simplified way, as follows:

$$g_{rq} = \left(\frac{d_{qq}}{d_{rq}}\right)^{\alpha}.$$

The constant α is the path loss exponent and its value depends on the propagation media. Finally, d_{qq} and d_{rq} are the distance, both in units of length, from UE_q to BS_q and from UE_r to BS_q, respectively.

As for estimations carried out in support of the model considered herein, the system model uses an initial estimation step in order to obtain $\mathbf{H}_{qq}$ (and optionally $\mathbf{H}_{rq}$) at the q-th BS. We consider perfect estimation of $\mathbf{H}_{qq}$, whereas estimation errors may be considered for $\mathbf{H}_{rq}$. The signaling load is not taken into account. That is, it is expected that a previous step is performed so that all this information is obtained.

For each UE, the average transmit power is constant and given by

$$\mathbb{E}\left\{\|\mathbf{x}_q\|^2\right\} = \frac{1}{N}\mathrm{tr}\left(\mathbf{F}_q\mathbf{F}_q^H\right) = P_q, \quad \forall q \in \Gamma, \tag{2.4}$$

where P_q is the average transmitted power in units of energy per signaling period. Also, the symbols are assumed to be uncorrelated, which means that $\mathbb{E}\left\{\mathrm{s}_q\mathrm{s}_q^H\right\} = \mathbf{I}$.

At each receiver (e.g., at the receiver of each UE), the MUI is treated as additive noise. This assumption is due to the fact that interference cancellation algorithms need some information (e.g., CSI) from interfering users [20] increasing the system signaling load. Hence, the estimated symbol vector at the q-th BS is defined as

$$\hat{\mathrm{s}}_q = \mathrm{D}\left[\mathbf{G}_q^H\mathbf{y}_q\right], \tag{2.5}$$

where $\mathbf{G}_q$ represents the minimum mean-square error (MMSE) stage [11, 21] and it is defined as

$$\mathbf{G}_q = \mathbf{R}_{-q}^{-1}\mathbf{H}_{qq}\mathbf{F}_q\left(\mathbf{I} + \mathbf{F}_q^H\mathbf{H}_{qq}^H\mathbf{R}_{-q}^{-1}\mathbf{H}_{qq}\mathbf{F}_q\right)^{-1}, \tag{2.6}$$

where $\mathbf{R}_{-q}$ corresponds to the interference-plus-noise covariance matrix estimated by the q-th BS and the subscript $-q$ denotes all the players belonging to Γ except the q-th player. Also in [11, 21], this matrix $\mathbf{R}_{-q}$ is defined as

$$\mathbf{R}_{-q} \triangleq N_o\mathbf{I} + \sum_{r\neq q}|g_r|\mathbf{H}_{rq}\mathbf{F}_r\mathbf{F}_r^H\mathbf{H}_{rq}^H$$

which is clearly a function of the interfering signals.

Before transmitting, each UE selects a precoding matrix $\mathbf{F}$, which is related to an antenna subset. Generally, for a given UE, the selection of $\mathbf{F}$ is based on some information fed back by the BS with which the UE is associated, as illustrated in Fig. 2.4.

In particular, Fig. 2.4 illustrates example transmitter circuits that may be included in one or more of the UEs, introduced in Fig. 2.1. The circuitry includes a multiplexer, RF modulators, RF switching circuits, a number of M_T transmit antennas and a precoding matrix selection circuit. In operation, symbols are multiplexed into a number of N streams, each of which is modulated by one of the RF modulators. The modulated stream(s) is (are) input into the RF switching circuit, where they are applied with particular weights to each of the transmit antennas, according to a precoding matrix selection, as made by the precoding matrix selection circuit. It is worth mentioning that $M_T \geq N$. The resultant uplink MIMO stream(s) is (are) transmitted through an uplink propagation channel $\mathbf{H}$ and precoding matrix

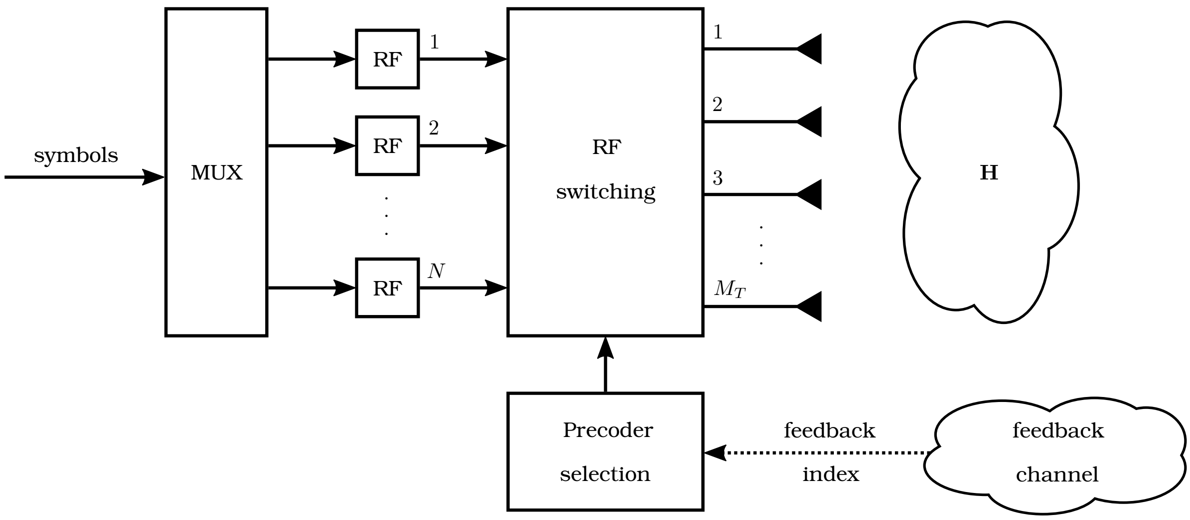

Figure 2.4: Block diagram of a user equipment.

selection feedback is received through a donwlink feedback propagation channel.

Consider a codebook $\mathcal{W}$ as being the set of all precoding matrices available for every entity in the system (e.g., for all UEs). For purposes of antenna subset selection, one may define each element of $\mathcal{W}$ as an $M_T \times N$ submatrix of an identity matrix $\mathbf{I}$. That is, the unique non-null entry of each column of this submatrix selects a transmit antenna. In order to index the elements of $\mathcal{W}$, assume an index set $\mathcal{I}$ defined as

$$\mathcal{I} \triangleq \{1, 2, \dots, \binom{M_T}{N}\}.$$

Thus, a bijective function $f : \mathcal{I} \leftrightarrow \mathcal{W}$ maps the elements of $\mathcal{I}$ onto the elements of $\mathcal{W}$ properly. For example, for $M_T = 3$ and $N = 2$:

$$\mathcal{W} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} \quad \mathcal{I} = \{1, 2, 3\}$$

$$f(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad f(2) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad f(3) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

For the sake of simplicity, it may be assumed that every receiver-transmitter pair has the same configuration, i.e., the same number of RF chains, and transmit and receiver antennas. Therefore, each receiver-transmitter pair works with the same codebook $\mathcal{W}$.

2.4 State-of-the-art

The most relevant works related to this Master's thesis can be divided into two categories: 1) single-user communication and 2) multi-cell and multi-user communication.

As for single-user communication, antenna selection for SM systems has been proposed in [5, 22], where the antenna selection criterion is the maximization of the minimum post-processing SNR at the BS. Besides, antenna selection for space-time block code (STBC) systems has been proposed in [23], where the antenna selection criterion is the maximization of the Frobenius norm of the channel matrix of the pair (UE,BS). However, these works do not take into account scenarios with interference which is a significant issue in real systems.

As for multi-cell and multi-user communication, multi-cell optimization for diversity multiple-input-single-output (MISO) schemes and interference mitigation has been studied in [24], where a game-theoretic framework is used for a 2-cell scenario in downlink communication and each BS aims at the maximization of its error probability making use of partial CSI. In addition to this, [25] has generalized antenna selection algorithms proposed in [22, 26] for interference limited MIMO wireless environments. In that work, the antenna selection criterion is the maximization of the post-processing SINR at each BS through a non-iterative algorithm. However, [24] does not consider MIMO configurations and [25] does not use an iterative algorithm in order to mitigate the inherent interference. The works [8, 11, 27] have applied game theory in multi-user MIMO systems. Those works contribute with a general game-theoretic framework using the IWF algorithm [28] to find out optimal precoding matrices for SM systems. Also, they derive sufficient condition (e.g., convex precoder set) ensuring existence and uniqueness of the Nash equilibrium. However, optimal precoding techniques are beyond the scope of this Master's thesis.

Based on those reference works, this Master's thesis aims at providing a game formulation for a SM system applying antenna subset selection in a environment with MUI. This work particularizes the general framework in [8, 11, 27] for a simpler one which performs antenna subset selection instead of optimal precoding. In the next chapter, this game formulation as well as the distributed algorithm will be described in detail.

Chapter 3

Game-theoRetic Antenna Subset Selection (GRASS)

3.1 Antenna Subset Selection Game

As proposed herein, precoder matrix selection game employs a game theory tool to solve the precoding selection problem, based on exploiting its interesting feature of solving optimization problems in a non-centralized way. For example, there is a defined set of precoder matrices available for use, wherein a matrix element value of “1” selects a corresponding antenna at the UE, for use in MIMO uplink transmission by the UE. Conversely, a matrix element value of “0” deselects the corresponding antenna. Thus, the particular precoding matrix selected for a given UE defines the particular subset of antennas used by that UE for MIMO transmission in the uplink.

Based on this approach, each BS in a set of BSs supporting a corresponding set of UEs that are co-channel interferers may be configured to play a game. According to the game, each BS uses the known (or indirectly estimated) precoder matrix selections made by the other BSs for their respective UEs, to estimate the covariance of interference and noise at the BS for its UE’s uplink signal. Each BS then uses that covariance estimate to determine the precoder matrix selection that optimizes in some sense the reception of its UE’s uplink signal.

For example, in a given round of game play, a given BS estimates the SINR for each (MIMO) stream received on the uplink from its associated UE, and determines the precoder matrix selection that maximizes the minimum one of the (per-stream) SINRs. Each BS in the overall set of BSs carries out the same selection processing for its associated UE, in the given round of game play. Game play thus advances to the next iteration with each BS updating its covariance estimate in view of the new precoder matrix selections. If there is a BSC in the system, such information is shared among the game-playing BSs through the BSC. Otherwise, each BS measures pilot or other reference signals, as transmitted by the interfering UEs using their newly selected precoder matrices.

More broadly, the contemplated game uses the fundamental model of game theory. The three key components of the game model include: (1) the set of players; (2) the set of actions; and (3) the set of objective functions.

Set of Players

In general, the players are the systemic entities that are able to act as rational decision-makers. They belong to the set of players which, in the game described herein, is the same set Γ defined in (Eq. 2.1). That is, the players are the same receiver-transmitter pairs $(\mathrm{UE}, \mathrm{BS})$ previously referred to as "neighboring links."

Set of Actions

For the q-th player, an action, drawn from the set of available actions $\mathcal{A}_q$, stands for the choice of some precoding matrix in $\mathcal{W}$, which means that

$$\mathcal{A}_q = \mathcal{W}, \quad \forall q \in \Gamma, \tag{3.1}$$

and the joint set of the action space of all players is the Cartesian product $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2 \times \cdots \times \mathcal{A}_Q$. In fact, this decision rule behind an action is called strategy. But which action a player will make depends on information available to that player. One may specify this information as being the interfering term inherent in the SINR expression, which will be described later. Once a player determines or otherwise obtains this information, that player will be able to make a decision following the player's (pure) strategy.

Set of Objective Functions

The outcomes of the game are represented by the output values of the objective (or utility) functions. Moreover, these functions must be choosen so that an action of a player somehow impacts the other players. For the nonzero-sum game contemplated herein, the q-th player observes a particular outcome (payoff) through its own utility function u_q after an action tuple made by all the players in a game iteration, such that

$$u_q : \mathcal{W}^Q \to \mathbb{R}, \; \forall q \in \Gamma. \tag{3.2}$$

It is worth noting that a given player need not be aware of the other players' utility functions, which turns the model into a game with incomplete information.

From the system model in Eq. (2.3), the SINR in the k-th data stream after the MMSE stage at the q-th BS is given by [11] as

$$\mathrm{SINR}_{k,q} = \frac{1}{\left[\left(\mathbf{I} + \mathbf{F}_q^H \mathbf{H}_{qq}^H \mathbf{R}_{-q}^{-1} \mathbf{H}_{qq} \mathbf{F}_q\right)^{-1}\right]_{k,k}} - 1 \tag{3.3}$$

with $\mathbf{R}_{-q} \triangleq (\mathbf{R}_r)_{r \neq q}$. From Eq. (3.3), one can see that there exist conflict of interests among the players, since $\mathbf{R}_{-q}$ is a function of the precoding matrices chosen by the interfering users. Thus, $\mathbf{R}_{-q}$ is the information that the q-th player has to realize at each game iteration. Hence, one may advantageously define the utility function of the q-th player as follows:

$$u_q(\mathbf{F}_q, \mathbf{R}_{-q}) = \min_k \mathrm{SINR}_{k,q}, \quad \forall k \in \mathcal{N}, \; \forall q \in \Gamma. \tag{3.4}$$

The motivation for optimizing the minimum SINR comes from the intuition that the performance of the receiver should improve as the smallest value of the SINR increases [5]. Here, the "smallest" SINR value is the minimum per-stream SINR, for the multi-stream MIMO uplink between a given BS playing the game, and its associated UE.

3.1.1 Game Formulation

The neighboring links were identified as being the contenders in the system. Therefore, one may consider each one a rational decision-maker, i.e., a player in the game. From the game standpoint, each player contends for the maximization of its own SINR. In practice, each player's strategy is to select one of the precoding matrices in $\mathcal{W}$ after determining or otherwise obtaining the information $\mathbf{R}_{-q}$ in a game iteration.

Let $\mathcal{G}_1$ be the non-cooperative and nonzero-sum game, which is written in normal form:

$$\mathcal{G}_1 = \left\langle \Gamma, \mathcal{W}^Q, \{u_{q\in\Gamma}\} \right\rangle,$$

where the first argument is the set of players, the second is the action space and the last one represents all individual utility functions. Stated in mathematical terms, $\mathcal{G}_1$ has the following structure:

$$(\mathcal{G}_1): \quad \begin{cases} \underset{\mathbf{F}_q}{\text{maximize}} & u_q(\mathbf{F}_q, \mathbf{R}_{-q}) \\ \text{subject to} & \mathbf{F}_q \in \mathcal{W}, \end{cases} \qquad \forall q \in \Gamma \tag{3.5}$$

where $\mathcal{W}$ is the codebook known by all the players. The manner in which the interference matrix $\mathbf{R}_{-q}$ is obtained depends on the distributive algorithm adopted, which is detailed later herein.

3.1.2 Game Solution: *Nash Equilibrium*

One may define the solution of the game $\mathcal{G}_1$ as being a pure-strategy NE. This kind of equilibrium is established if each player has chosen an action and no one can benefit by changing its action unilaterally while the others keep theirs unmodified [12]. It is considered a predicted outcome of the game in the sense that if that outcome is reached, then no player has an incentive to deviate from it. Therefore, an action tuple $\{\mathbf{F}_q^\star, \mathbf{F}_{-q}^\star\}$ is a NE if

$$u_q(\mathbf{F}_q^\star, \mathbf{R}_{-q}(\mathbf{F}_{-q}^\star)) \geq u_q(\mathbf{F}_q, \mathbf{R}_{-q}(\mathbf{F}_{-q}^\star)), \quad \forall \mathbf{F}_q \in \mathcal{W},\ \forall q \in \Gamma. \tag{3.6}$$

The superscript $\star$ denotes that the underlying precoder leads to a NE. The structure above is a convenient form for representing a NE [12].

In other words, an equilibrium point, a NE in this example, means that each UE will transmit with the antenna subset related to its precoding matrix according to the game result. But a particular NE action tuple does not say anything about how this equilibrium point is reached or about uniqueness. Actually, the BSs have limited information and therefore they can not predict an equilibrium. Thus, some "learning" algorithm must be concerned in order to dynamically reach an equilibrium. The process of reaching an equilibrium point is an important issue and it is usually described by a distributed algorithm. This work defines a distributed algorithm for antenna subset selection.

Thus far, sufficient conditions for the existence of a NE have not been identified. Despite the fact that the number of players is finite, this work did not adopt mixed-strategy solutions. From [8, 12, 29], some standard results from fixed-point theory and contraction maps are used to state these conditions for pure-strategy solutions. One requires a nonempty, convex and compact codebook $\mathcal{W}$ to guarantee the existence of at least one NE. However, the codebook design adopted in a real-world communication system does not necessarily satisfy such requirements. Hence, another antenna selection algorithm is made available in cases where equilibrium is not reached (e.g., within an allowed number of game iterations). For example, upon failure to reach equilibrium, a BS may fall back to using a non-iterative algorithm.

In particular, it is proposed herein to use the MMSV algorithm for precoding matrix selection in case there is no point of equilibrium. Use of the MMSV algorithm has been proposed in [5]. In applying the MMSV algorithm, the q-th BS, after acquiring the estimation of the channel matrix $\mathbf{H}_{qq}$, obtains the singular values of $\mathbf{H}_{qq}$ through a singular value decomposition (SVD). Then, it chooses the antenna subset of $\mathbf{H}_{qq}$ which yields the largest minimum singular value.

In point of fact, a player recognizes the lack of a NE through the use of a trial and error convergence method. That is, the player makes use of the direct application of (Eq. 3.6), hopping from one precoding matrix to another in order to find an equilibrium point. If no point of equilibrium is found after the check of all possible action tuples, the game is over unsuccessfully and each player switches to the MMSV algorithm for precoding matrix selection. Futher detail will be presented later. In simulations and/or empirical observations, it has been noted that a NE does not occur for some small number of channel realizations (less than 10%). Thus, in the approach adopted herein, a codebook $\mathcal{W}$ is used that is appropriate for the system at hand, despite the fact that it may not yield a NE for all channel realizations. In such cases, which are expected to be few in number, an alternative precoder matrix selection algorithm is used, such as MMSV. Of course, it is also contemplated that, for at least for some types of systems, the codebook $\mathcal{W}$ is designed to eliminate or at least greatly reduce the number of cases where a NE is not obtained. However, this issue is not addressed in this work.

3.2 GRASS Algorithm

The proposed distributed algorithm is designed for antenna subset selection, and is referred to as the Game-theoRetic Antenna Subset Selection (GRASS) algorithm. The GRASS algorithm is performed at each BS with no coordination among the UEs.

To better understand the GRASS algorithm, note that the broader MUI reduction game play involves a set of UEs that are operating as interferers with respect to one another, by virtue of reusing some or all of the same channel resources. Each such UE is supported by a given BS. That is, the game involves a set of neighboring (interfering) communication links, with each link formed as a receiver-transmitter pair between a supporting BS and its associated UE.

Now, for the GRASS context, the game action undertaken by each BS playing the game is an antenna subset selection, to be used by its associated UE. After an initial step of channel estimation, each BS is able to play the game $\mathcal{G}_1$. But each BS needs to determine some information from its set of interfering users in order to make rational decisions. In one approach, each BS may be provided with the needed information explicitly. Alternatively, each BS may estimate such information, e.g., derive it from measurements, etc. For example, game play involves an iterative exchange of information among the involved BSs until reaching a point of equilibrium—such an exchange may be conducted through a centralized BSC.

For the sake of simplicity, one may assume perfect channel estimation and an error-free link among BSs and between each BS and UE. Consequently, if there exists a NE point, the system ideally always converges to it. As long as these assumptions hold, the performance of the algorithm in terms of bit error rate (BER) does not depends on how the information exchange is performed. Of course, in practice, errors in the exchange of information between game players as well as estimation errors may degrade the performance of the game algorithm.

The flowchart showing the key processes of the algorithm as implemented at a game playing BS is depicted in Fig. 3.1. The block *game iteration* is the core of the algorithm and will be discussed in detail later. For now, one can see that a loop *counter* controls the game iterations and it is upper-bounded by the constant λ, defined as follows:

$$\lambda = \left[\binom{M_T}{N}\right]^Q. \tag{3.7}$$

Table 3.1: Summary of the GRASS algorithm.

Initial:	estimation step at each BS
Iterations:	BSs exchange information BSs try to reach a NE point $\mathcal{G}_1$ is played until all users converge
End:	BSs send the index $i^\star$ back to their users

In fact, the value of λ equals the number of all possible action tuples. This is not optimal because this value may be much larger than the number of iterations needed to reach an equilibrium point. However, the optimality of the parameter λ is not addressed herein and it works only as an upper-bound.

Furthermore, the block MMSV is triggered if and only if no point of equilibrium is found in λ iterations. Finally, the block *index feedback* is the last process. Through the limited-feedback link—i.e., the downlink—each BS sends to its associated UE the index of the precoding matrix related to the NE action. Then, the GRASS algorithm ends and each UE selects an antenna subset based on the index just provided to it by its BS.

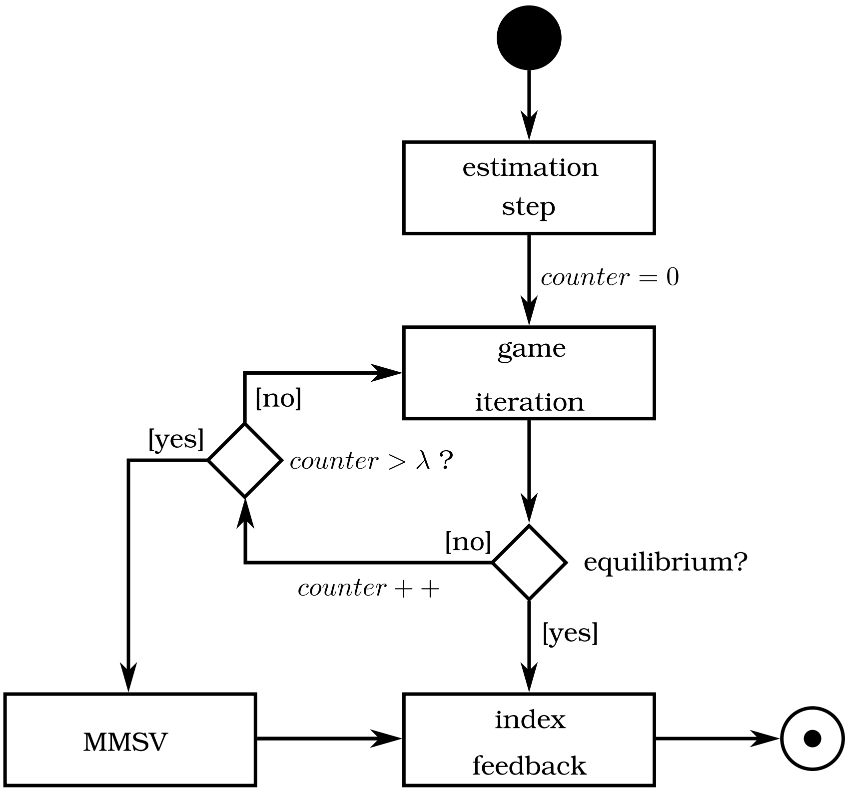

Figure 3.1: GRASS flowchart.

The algorithm may be summarized (Table 3.1) as: (1) performing an initial step of channel estimation at BS; and, (2) in each of a bounded number of iterations, the BSs exchange information about the precoder matrix selection made for their respective UEs, with each BS trying to reach the NE point, and with game play continuing until all BSs converge (or until a limited number of iterations is reached). The finalized precoding matrix selection arrived at by each BS is sent to the UE associated with that BS. Thus, Fig. 3.1 includes the above-described estimation step, a game iteration step, an equilibrium check step, a precoding matrix index selection feedback step, a counter check step, and an alternative precoding matrix selection step (e.g., MMSV algorithm).

In the following, two approaches on how the game iterations may be carried out will be described.

3.2.1 Game Iteration with Configuration 1

In this first approach, conveniently called "GRASS with configuration 1", a BSC supports game iteration. All the BSs playing the game for a given set of intercell-interfering UEs exchange information (through the BSC) in order to reach a NE. Also, those BSs require the knowledge of all the interfering channels—i.e., the q-th BS needs to estimate all the interfering channel matrices (i.e., $\mathbf{H}_{rq}$, for all r) in some previous step. First, BSs play $\mathcal{G}_1$ considering an initial action index tuple[1], for instance $(i_q[n], \mathbf{i}_{-q}[n])|_{n=0} = (1, \vec{\mathbf{1}})$. Here, the argument n means the stage domain and action index tuple is defined such that

$$i_q = f(\mathbf{F}_q), \quad i_q \in \mathcal{I}, \; \mathbf{F}_q \in \mathcal{W},$$

is the q-th action index which is an output of the bijective function f, defined in Section 2.3, and

$$\mathbf{i}_{-q} = \begin{bmatrix} i_1 & i_2 & \cdots & i_{q-1} & i_{q+1} & \cdots & i_Q \end{bmatrix}$$

is the related action index vector. At the stage $n+1$, the q-th BS generates an action message vector $\mathbf{m}_q$, which is the string of $b = \left\lceil \log_2 \binom{M_T}{N} \right\rceil$ bits representing i_q. After that, the BSC receives all the action message vectors from all the BSs through the direct wired links simultaneously. Then, it assembles a number of Q message vectors such that, for the q-th BS,

$$\mathbf{m}_{-q} = \begin{bmatrix} m_1 & m_2 & \cdots & m_{q-1} & m_{q+1} & \cdots & m_Q \end{bmatrix}.$$

Finally, the BSC sends back each message vector to the BS associated through the respective reverse wired link. Fig. 3.2 illustrates a numerical example of such processing, regardless the time domain. In this example, there are three BSs ($Q = 3$) exchanging information through the BSC. Considering $M_T = 3$ and $N = 2$, each action message vector $\mathbf{m}$ (i.e., $\mathbf{m}_1$, $\mathbf{m}_2$ and $\mathbf{m}_3$) has 2 bits.

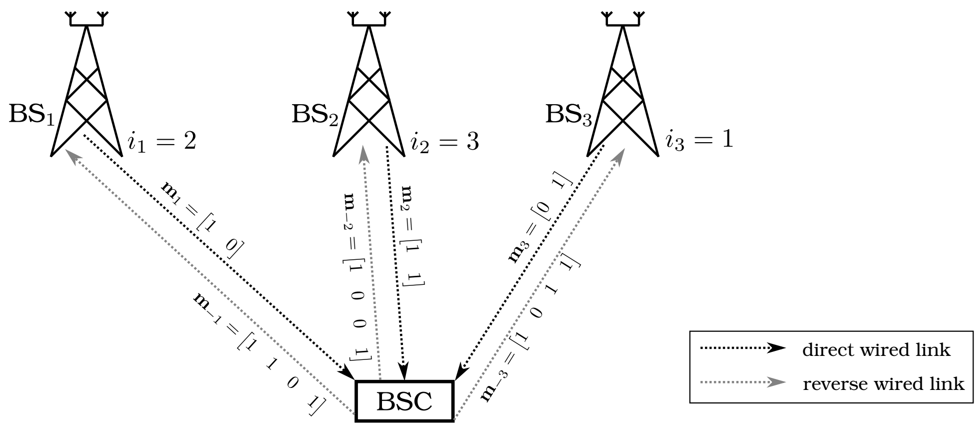

Figure 3.2: Example of information exchange via BSC.

Moreover, Fig. 3.3 illustrates the time diagram of a general example of such processing, for a given iteration. In this example, each action index i (i.e., i_q and i_r) is updated after a player action, whereas each action index vector $\mathbf{i}$ (i.e., $\mathbf{i}_{-q}$ and $\mathbf{i}_{-r}$) is updated after the BSC sends the message vectors back.

[1]The term $\vec{\mathbf{1}}$ stands for a vector whose entries equal one.

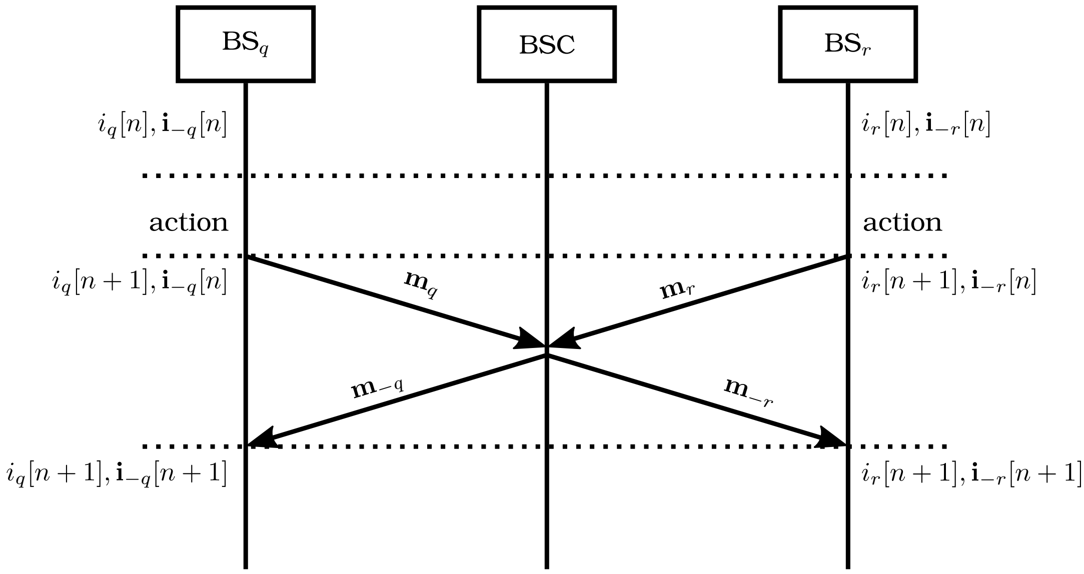

Figure 3.3: Iterations of the game via BSC.

3.2.2 Game Iteration with Configuration 2

In this other approach, conveniently called "GRASS with configuration 2", each BS exchanges information only with its associated UE. That is, the entity BSC is not necessary to enable the game $\mathcal{G}_1$ to be played—i.e., the set of BSs can play the game without the need for a centralized entity for exchanging certain game-play information among the BSs. Also, the BSs do not need to estimate all the interfering channels in some previous step. However, such an approach requires an extra estimation step in each iteration of game play. Each such iteration is depicted by way of general example in Fig. 3.4. First, each UE involved in the game transmits a pilot signal considering also an initial action index—i.e., a precoding matrix selection. Then, each BS, by knowing the initial action of its associated UE, draws the joint action of the others implicitly from an estimation of the matrix $\mathbf{R}_{-q}$ denoted by $\hat{\mathbf{R}}_{-q}$.

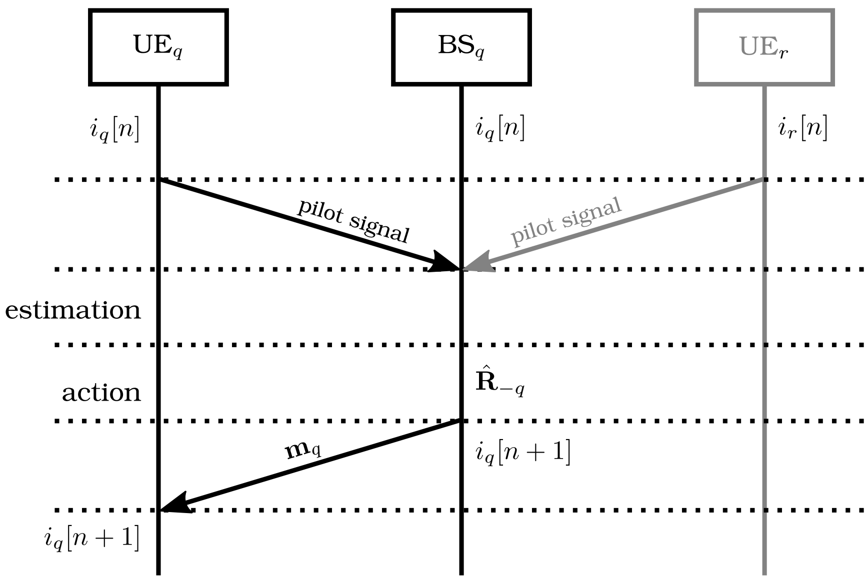

Figure 3.4: Iterations of the game via information estimation.

In other words, without the benefit of information sharing through a BSC or another entity, each BS playing the game can nonetheless estimate or otherwise infer the precoding matrix

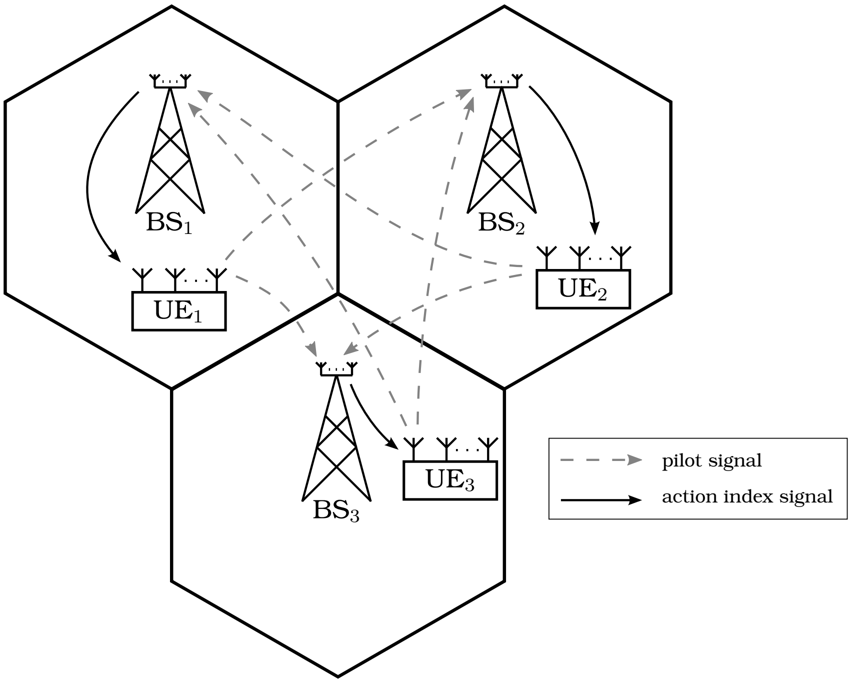

Figure 3.5: Example of information exchange without BSC.

selections made by the other BSs for their respective UEs, based on evaluating pilot signals from those other UEs. Subsequently, each BS plays $\mathcal{G}_1$ and generate the next action index. The stage $n+1$ is such that each BS sends back the next action index to its associated UE through the limited-feedback link. That is, the q-th BS generates the message vector $\mathbf{m}_q$ and sends it to the q-th UE. Fig. 3.5 illustrates an example of such processing showing the signals involved. In this example, there are three BSs (i.e., BS$_1$, BS$_2$ and BS$_3$) and their respective associated UEs.

As for estimation, one may consider an estimation error associated to the estimation step based on pilot signal evaluation. Assuming MMSE estimation [30, 31], the estimation error $\mathbf{E}_{rq}$, i.e., the error associated to the channel between the pair $(\mathrm{UE}_r, \mathrm{BS}_q)$, is denoted by

$$\mathbf{E}_{rq} = \mathbf{H}_{rq}\mathbf{F}_r - \hat{\mathbf{H}}_{rq},$$

where $\hat{\mathbf{H}}_{rq}$ is the MMSE estimate. The entries of $\mathbf{E}_{rq}$ are complex Gaussian distributed random variables with variance $\sigma^2_{\mathbf{E}_{rq}}$. Since the interfering channels are weighted by the path loss factors associated, the variance $\sigma^2_{\mathbf{E}_{rq}}$ is a function of the interference-to-noise ratio (INR) between each pair $(\mathrm{UE}_r, \mathrm{BS}_q)$, defined as

$$\mathrm{INR}_{rq} = \frac{g_{rq}}{N_o}.$$

Thus, $\sigma^2_{\mathbf{E}_{rq}}$ is conveniently defined as

$$\sigma^2_{\mathbf{E}_{rq}} = \frac{1}{1 + T_p \mathrm{INR}_{rq}}, \tag{3.8}$$

where T_p is the number of pilot symbols. However, how many pilot signals [30] must be used or even how exactly the estimation processing [31] is performed is beyond the scope of this work. It is worth noting that the game result is directly degraded as the variance $\sigma^2_{\mathbf{E}_{rq}}$ increases.

Table 3.2: Amount of bits per iteration.

Game iteration	link	# bits
via BSC	direct wired reverse wired	b $(Q-1)b$
via Information Estimation	limited-feedback	b

In other words, the q-th BS infers the actions made by the other BSs by evaluating the modified version of the interference-plus-noise covariance matrix denoted by $\hat{\mathbf{R}}_{-q}$, and defined as

$$\hat{\mathbf{R}}_{-q} = N_o\mathbf{I} + \sum_{r \neq q} \hat{\mathbf{H}}_{rq}\hat{\mathbf{H}}_{rq}^H.$$

Therefore, every estimate $\hat{\mathbf{H}}_{rq}$ contributes to the degradation of the information used by BSs in each game iteration.

Scalability Issue

One may assume a constant value for the number of RFs N. Then, two parameters of the system that are relevant to scalability are Q and M_T. Both of them imply an increase in the amount of information exchanged. Also, the way the game iteration is performed determines exactly how many bits are exchanged per iteration, as can be seen in Table 3.2. For example, in each game iteration, the number of bits exchanged via the BSC for the direct wired link is b, and $(Q-1)b$ for the reverse wired link. Further, b bits are exchanged for information estimation on the limited-feedback link. Fig. 3.6 graphically illustrates that the BSC-based approach demands a larger number of bits than the alternative approach that omits the BSC. Another important issue is the number of iterations needed to reach a NE point. That number depends on the channel condition, and thus varies. However, an upper-bound on the game play iterations is assumed, such as λ. Further, the configuration of each transceiver can easily be fixed, whereas the number of active mobile terminals, i.e., UEs, has to be flexible. Therefore, the value Q is determinant to evaluate the feasibility of the system in terms of the amount of information exchanged.

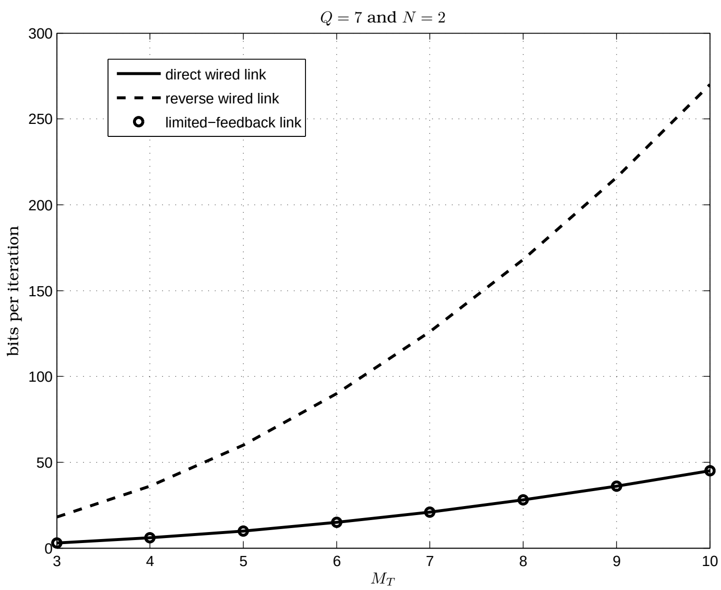

Figure 3.6: Bits per game iteration.

How efficient is a player in the game?

As the number of players Q increases, the positioning of the UEs may lead to some special configurations. Some UEs may be close to their respective BSs, which leads to a configuration such that those BSs observe low interference levels due to small pathloss. Those BSs do not contribute to the game because the conflict of interests between those BSs and the other BSs is potentially weak, even though those BSs remain to be a source of interference. To exploit this, a player removal procedure can be performed for those BSs which observe low interference levels.

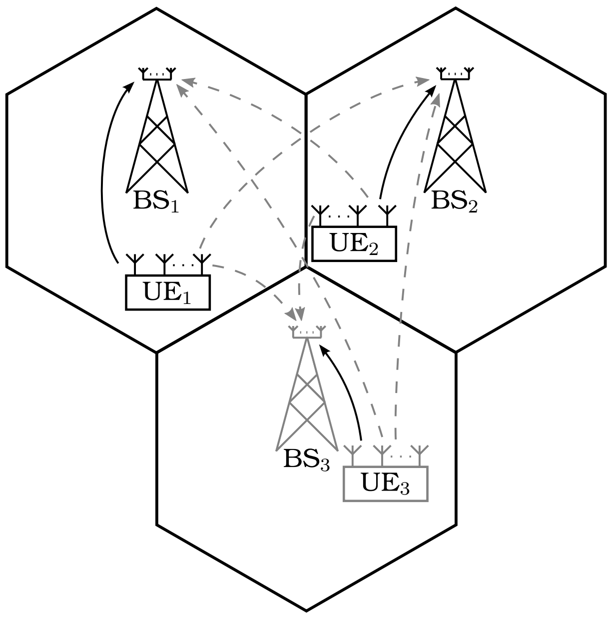

Figure 3.7: Typical configuration suitable for player removal procedure.

Accordingly, those BSs may directly use a MMSV algorithm for precoding matrix selection, whereas the other BSs remain to play the game iteratively. Fig. 3.7 illustrates such a configuration. This may lead to a worse solution compared to the game result with all BSs playing the game. But if one or more players are removed from the game play, the number of iterations needed to reach a point of equilibrium (if there is one) decreases. In addition to this, the estimation error associated with the covariance matrix increases as the INR decreases. Therefore, this trade-off motivates the use of this player removal procedure in order to avoid worst-case situations (e.g., high estimation error level and large number of iterations needed).

3.2.3 Summary

This chapter described the game model adopted in this Master's thesis formulating a non-cooperative static game. Each pair (BS, UE) acts as a player in such a game and the concept of NE point is considered to be the game solution.

Furthermore, the distributed algorithm proposed, i.e., the GRASS algorithm, is described in detail. There are two ways to implement such algorithm iteratively: (1) using a BSC (GRASS with configuration 1) and (2) without using a BSC (GRASS with configuration 2). Finally, the scalabity issue was discussed and the conclusion is that the parameter Q is determinant to evaluate the feasibility of the system.

In the next chapter, a few simulation results will be shown in order to analyze the performance of the GRASS algorithm.

Chapter 4

Simulation Results

Simulation results for game play as contemplated herein for MUI reduction are based on evaluating the BER, averaged over at least 10^6 channel realizations, against the transmit SNR via Monte Carlo simulations. A binary phase shift keying (BPSK) modulation was used, as well as a data block length $L = 102$ symbols[1] in each transmission setup. Also, channel realizations are independent identically distributed (i.i.d) from block to block. Here, the structure $(M_T, N) \times M_R$ means that the system selects N transmit antennas out of M_T and receives the transmitted signal with M_R antennas. Also, one may define SIR at each BS as being

$$\mathrm{SIR}_q = \left(\sum_{r=1, r\neq q}^{Q} g_{rq} \right)^{-1}, \quad \forall q \in \Gamma. \tag{4.1}$$

The analysis considers a scenario with only two users, i.e., 2 UEs, with varying SIR values observed at each BS. The algorithms used as reference cases are the MMSV proposed in [5], which chooses the antenna subset that yields the channel with largest minimum singular value, and the exhaustive search, which is used as a performance bound. In view of showing the diversity gain due to the antenna selection process [6], some curves of SM MIMO scheme with no antenna selection[2] are shown. Additional results consider five types of 7-cell scenario, in which every BS observes a different SIR, as well as further measurements such as the number of game iterations and the probability of reaching a NE. For these scenarios, both GRASS configuration 1 and GRASS configuration 2 were considered.

4.1 Scenario with 2 cells

In more detail for an example 2-cell scenario, there are two adjacent cells and, consequently, two neighboring links. The UEs are positioned such that each BS observes the same SIR. Because the UEs are symmetrically positioned, they have the same performance in terms of BER and $\mathrm{SIR}_1 = \mathrm{SIR}_2 = \mathrm{SIR}$. Thus, it is enough to illustrate only the average BER curves. Fig. 4.1 illustrates this type of scenario.

In Fig. 4.2, the GRASS algorithm has a performance loss compared to the lower bound represented by the (computationally expensive) exhaustive search. It is worth noting that the lower bound curve is drawn from a centralized algorithm that yields an optimal performance, whereas the GRASS algorithm is suboptimal. However, the GRASS algorithm

[1]The number of symbols must be a multiple of N due to the fact that the symbols are spatially multiplexed through N selected antennas. For this example discussion, it is assumed that the parameter N ranges from 2 to 3. Therefore, one may choose the value 102 as a multiple of these values. Of course, the length L may be any multiple of N.

[2]In this scheme, each receiver-transmitter pair has N transmit antennas, N RF chains and M_R receive antennas.

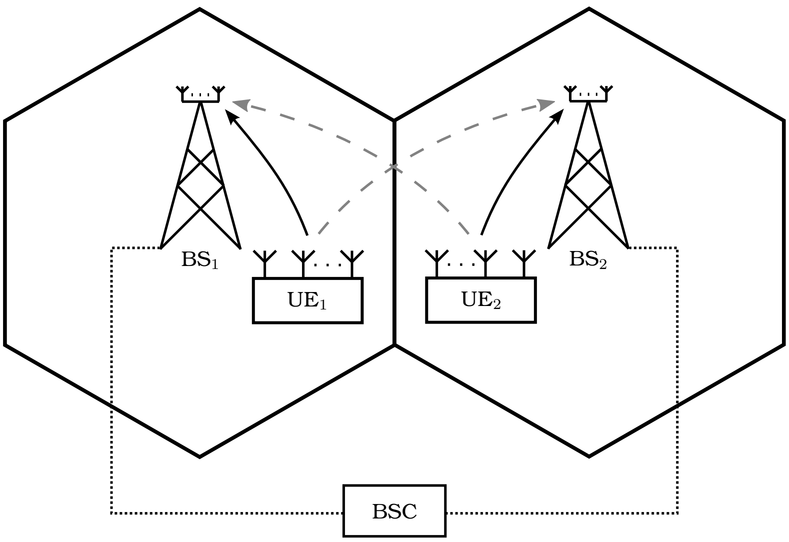

Figure 4.1: Example of 2 UEs symmetrically positioned in a 2-cell scenario.

provides for a decentralized approach, which offers significant advantages when used in a wireless communication. Besides that significant advantage, the performance of the GRASS algorithm is significantly close to the optimal. For a BER target equal to 10^{-2}, the penalty is approximately 1.3 dB.

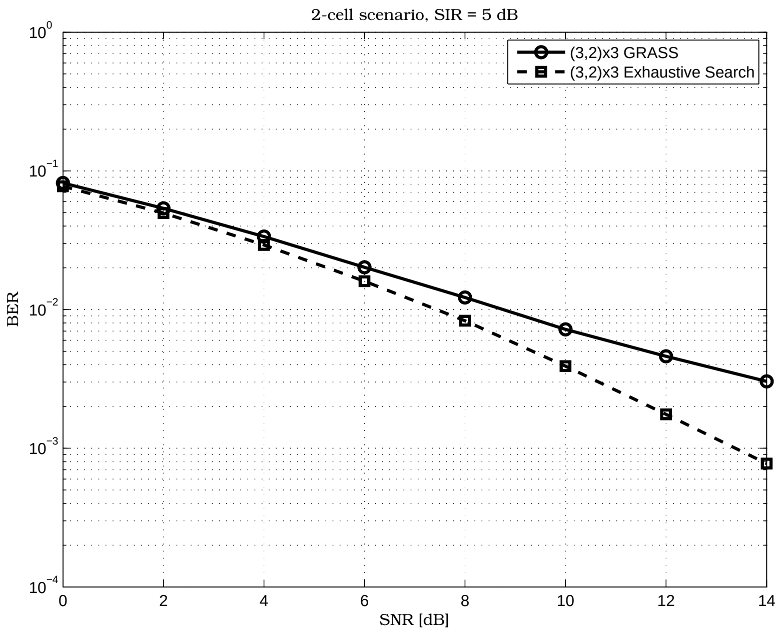

Figure 4.2: Bit Error Rate - $(3, 2) \times 3$ System.

In Figs. 4.3 and 4.4, one sees that the proposed game approach—the use of GRASS—achieves a lower BER floor compared to MMSV. This performance advantage arises because the GRASS algorithm inherently mitigates MUI. Besides, the diversity gain obtained by increasing the number of transmit antennas from 3 to 4 is observed because the slope of both MMSV and GRASS curves in Fig. 4.4 is steeper than the slope of the respective curves in Fig. 4.3.

On the other hand, the conflict aspect of the game diminishes as MUI decreases. That is, there is no significant mutual interference between the links in high SIR regimes. Therefore, the game solution approaches the reference single user case in [5]. This behavior can be

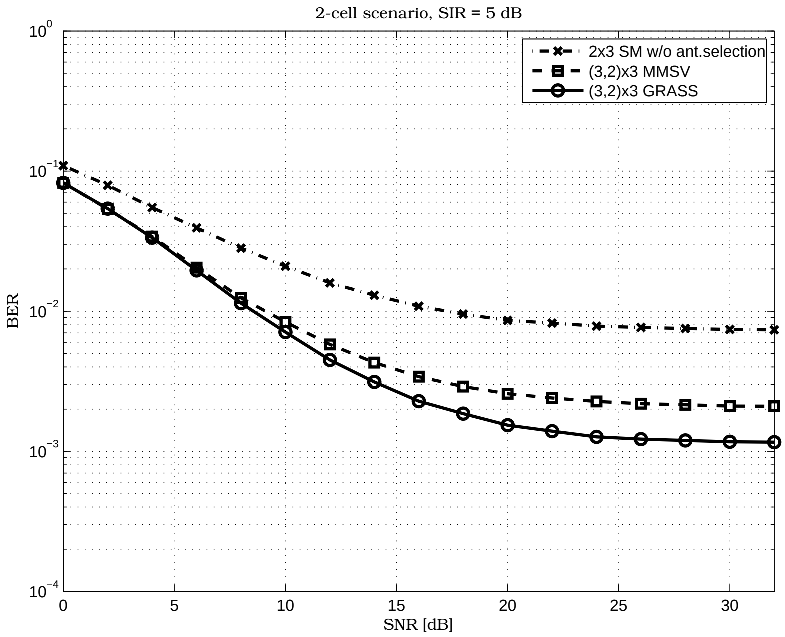

Figure 4.3: Bit Error Rate - $(3, 2) \times 3$ System.

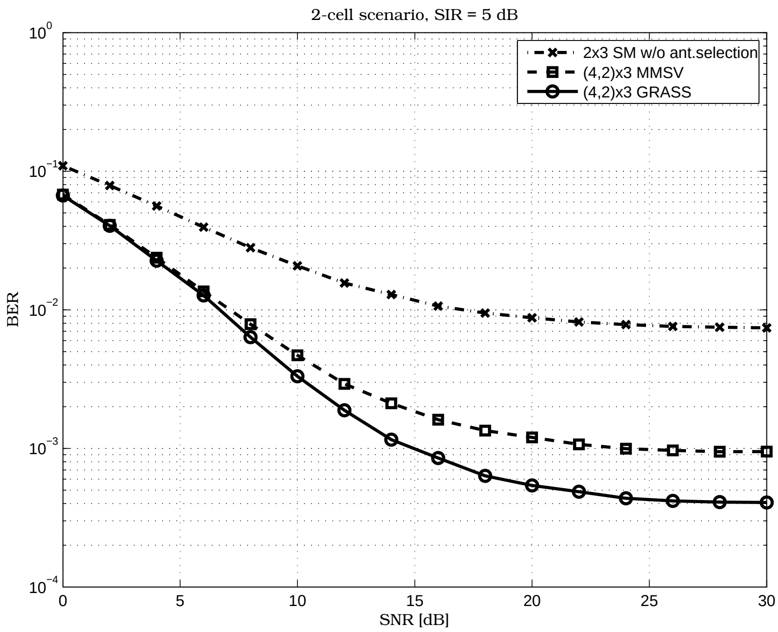

Figure 4.4: Bit Error Rate - $(4, 2) \times 3$ System.

seen in Figs. 4.5 and 4.6. In the former, the obtained performance gain is lower compared to Fig. 4.3, whereas in the latter the GRASS curve has almost no gain compared to the MMSV curve.

In Figs. 4.7 and 4.8, the GRASS curve does not outperfom significantly the MMSV curve, because the number of receive antennas is not larger than the number of RF chains. In other words, there is not enough diversity (or degrees of freedom) at the receivers to cancel the MUI and both algorithms perform relatively poorly with a BER floor of $3 \cdot 10^{-2}$, approximately.

For all the configurations above, a BSC was considered in order to perform information exchange (game iteration via BSC). That is, none of the BSs observed performance loss due to estimation errors.

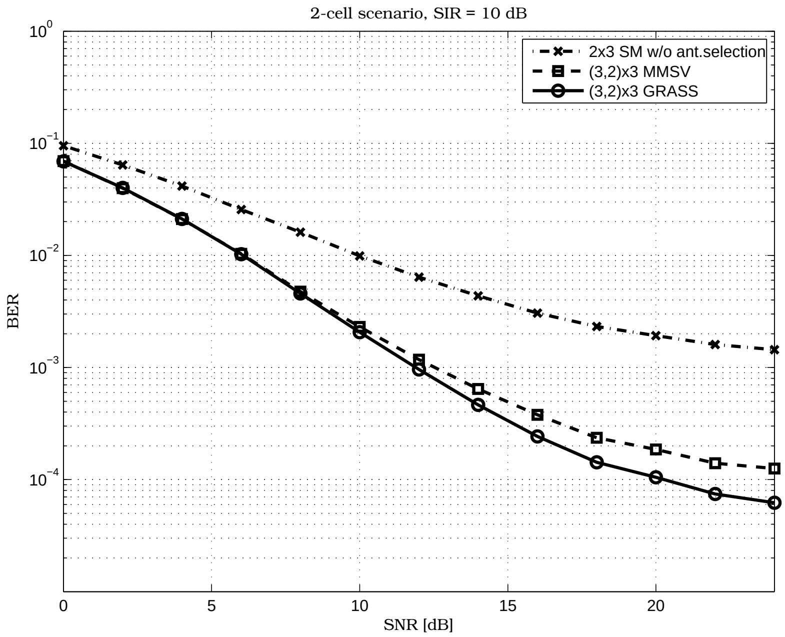

Figure 4.5: Bit Error Rate - $(3, 2) \times 3$ System.

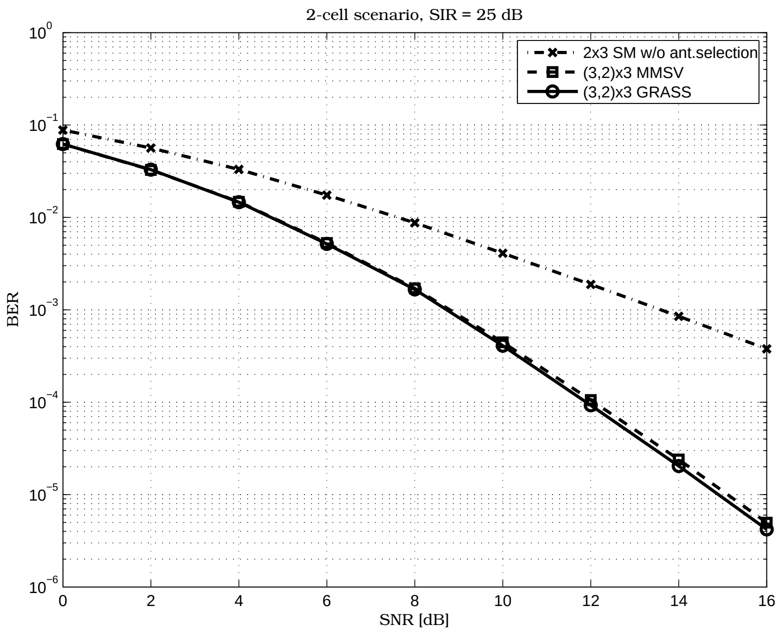

Figure 4.6: Bit Error Rate - $(3, 2) \times 3$ System.

4.2 Scenario with 7 cells

In a seven-cell scenario, there are 1 central cell, i.e., BS_1, and 6 surrounding ones. Consequently, there are 7 neighboring UEs. Each cell has a radius R in meters. That is, for this basic scenario, a first BS in a central cell supports a corresponding UE, where that UE is an interferer with respect to the radio links between six other neighboring UEs, each in one of the surrounding six cells and supported by the BS in that cell. As such, there are seven mutually interfering links, each link comprising a receiver/transmitter (BS/UE) pair. For this scenario and transceiver configuration, the upper-bound λ equals 2187 iterations.

With $Q = 7$, it is difficult to find symmetric user positions in the cells such that every BS observes the same SIR. Therefore, one may define five types of scenarios in which each UE has

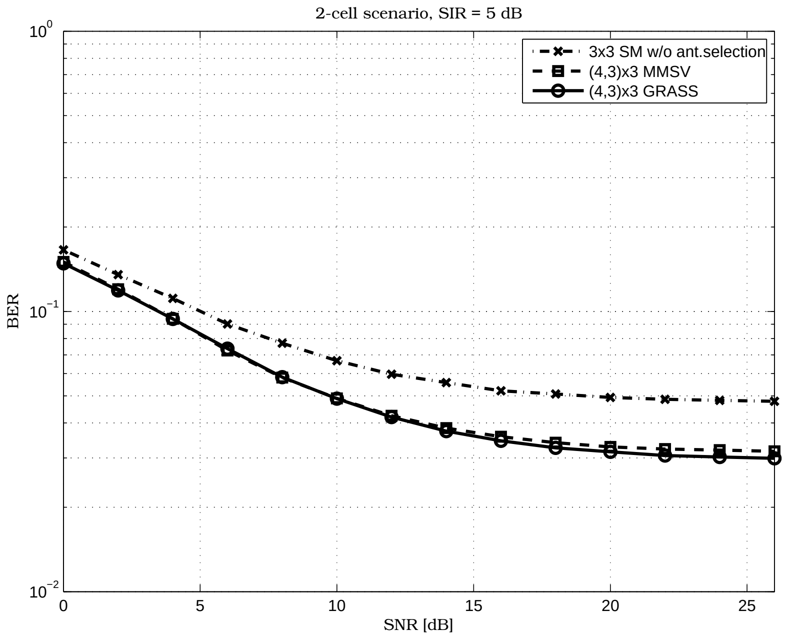

Figure 4.7: Bit Error Rate - $(4, 3) \times 3$ System.

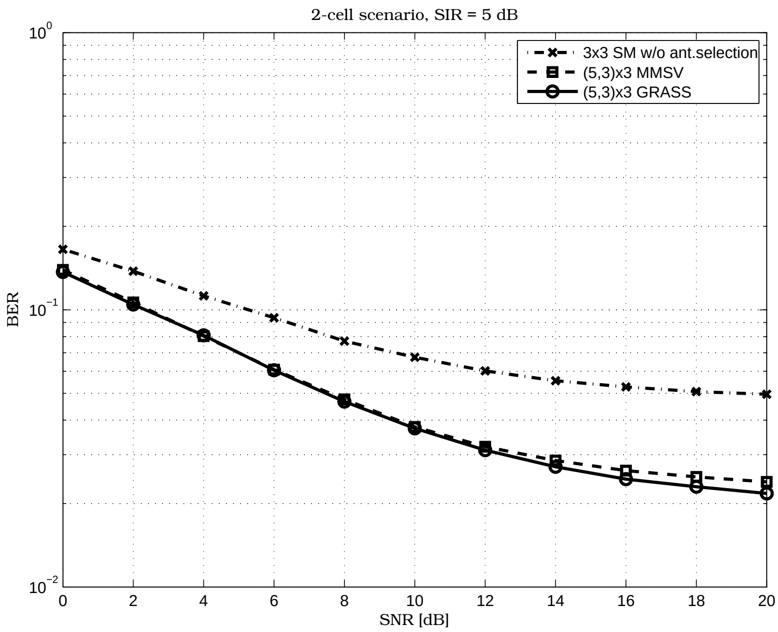

Figure 4.8: Bit Error Rate - $(5, 3) \times 3$ System.

different SIR levels. Each scenario is described (in terms of SIR) in Table 4.1. The parameters R and α equal 500 meters and 3.76, respectively. For each type of scenario, the position of each UE was set at random inside its cell. The values showed in Table 4.1 were obtained after a few attempts such that each type of scenario had its own characteristic. Moreover, one may evaluate the system performance in terms of average BER, best-user BER (i.e., the UE with the highest SIR level) and worst-user BER (i.e., the UE with the lowest SIR level). Both GRASS configurations are considered. For GRASS configuration 2, the parameter T_P equals 72.

It is evident that the GRASS approach with configuration 1 (i.e., with the use of a BSC) always outperforms the MMSV algorithm independently of the scenario type, which can be seen in Figs. 4.9 through 4.13. However, the magnitude of this performance gain depends on the SIR level of each UE. On the other hand, GRASS with configuration 2 (i.e., without

Table 4.1: SIR [dB] per UE in each 7-cell scenario

Scenario	Description	SIR_1	SIR_2	SIR_3	SIR_4	SIR_5	SIR_6	SIR_7
type 1	high SIR levels	15.93	11.47	32.30	31.30	33.49	30.09	22.72
type 2	only one high SIR level	22.63	7.62	7.57	7.88	8.11	8.13	7.94
type 3	only one low SIR level	3.99	25.69	35.20	19.02	20.83	28.29	25.52
type 4	low and high SIR levels	5.64	22.56	34.60	15.10	8.48	8.40	6.58
type 5	low SIR levels	2.28	6.12	4.89	6.61	7.31	7.41	7.20

BSC) undergoes significant performance loss due to the estimation error associated. This loss implies that the GRASS algorithm with such configuration outperforms the MMSV for only some SNR ranges.

GRASS with configuration 1

For GRASS with a BSC, from the curves, one may notice that if the SIR level is lower than 5 dB, the gain is small because the MUI is strong and the GRASS algorithm does not manage to satisfactorily mitigate the interference. This behavior can be seen in Figs. 4.11 (BS_1 in type 3) and 4.13 (BS_1 in type 5) for the worst-user case. One might also notice that the gain is quite small for high SIR levels (higher than 20 dB). This small gain may result because the MUI is small for such cases, which means that the conflict of interest among the users becomes small. This behavior can be seen in Figs. 4.9 through 4.12 (BS_5 in type 1, BS_1 in type 2, BS_3 in type 3, and BS_3 in type 4) for the best-user case.

On the other hand, the gain advantages become significant as the SIR levels range from 5 dB to 20 dB. For this SIR range, the conflict aspect of the proposed game-based approach is significant, and carrying out the game thus provides significant MUI reduction. See, for example, Figs. 4.9 (BS_2 in type 1), 4.10 (BS_3 in type 2) and 4.12 (BS_1 in type 4) for the worst-user case, as well as in Fig. 4.13 (BS_6 in type 5) for the best-user case.

Another aspect is the average behavior of the system in terms of BER, in which the gain is averaged over the individual gains obtained by each user. Thus, the SIR levels of the UEs reflect on the average BER directly. We see that the average gain does not appear to be significant in Fig. 4.11 since the SIR_1 is small and the remaining SIR levels are high in scenario type 3. Throughout the other scenario types, there is a mixture of intermediate SIR levels with both high and low levels, which provides a considerable average gain.

GRASS with configuration 2

As for this other configuration (i.e., with no BSC), it is evident that the GRASS algorithm has a poor performance in low INR regime. In other words, when the interference level observed by a BS is low, the estimates of the interfering channels are degraded. From Eq. (3.8), it is clear that small values of INR yield a large value of the variance $\sigma^2_{\mathbf{E}_{rq}}$. For example, Fig. 4.9 shows the worst performance of such a configuration. All the BSs observe low INR (i.e., high SIR levels) such that the solution provided by this configuration is outperformed by both the others. The same behavior can be seen in Fig. 4.10 for the best-user case (BS_1 in type 2) and in Figs. 4.11 and 4.12 for both best-user (BS_3 in type 3 and BS_3 in type 4) and average cases.

On the other hand, when BSs observe high INR levels, the estimation error associated with the interfering matrices becomes small. Thus, as the noise power decreases, GRASS with such configuration tends to have the same performance of GRASS with configuration 1. For example, in Fig. 4.10, GRASS with configuration 2 converges to GRASS with configuration 1 for both worst-user (BS_3 in type 2) and average cases for SNR at 24 dB. In Figs. 4.11 and 4.12, the same behavior occurs for the worst-user case (BS_1 in type 3 and BS_1 in type 4) for SNR at 24 dB and 22 dB, respectively. Also, in Fig. 4.13 it is evident the good performance of such configuration in scenario type 5 (high INR levels at all BSs) because the curves show GRASS with configuration 2 converging to GRASS with configuration 1 for all cases.

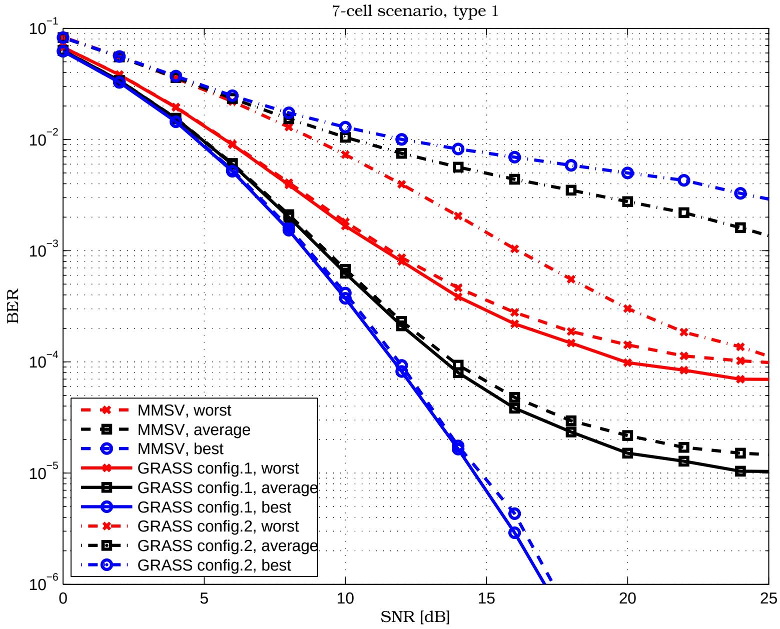

Figure 4.9: Bit Error Rate - $(3, 2) \times 3$ System.

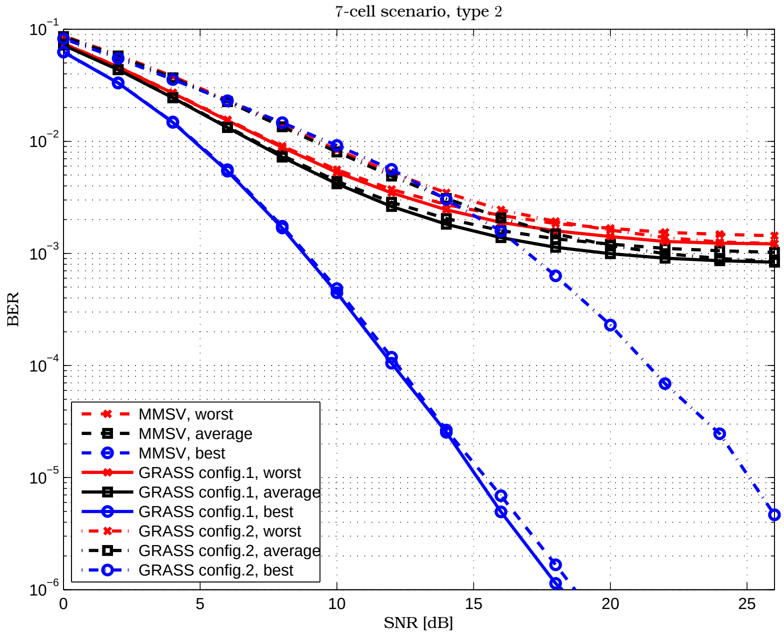

Figure 4.10: Bit Error Rate - $(3, 2) \times 3$ System.

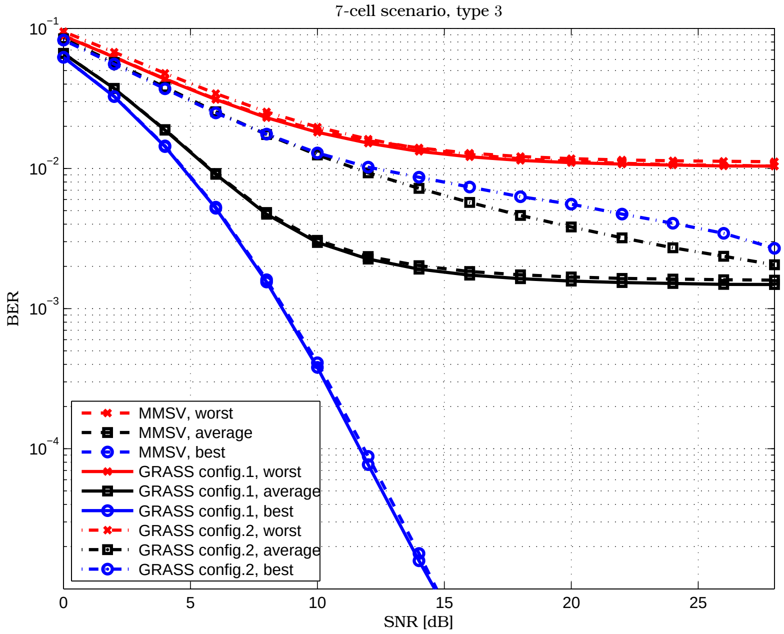

Figure 4.11: Bit Error Rate - $(3, 2) \times 3$ System.

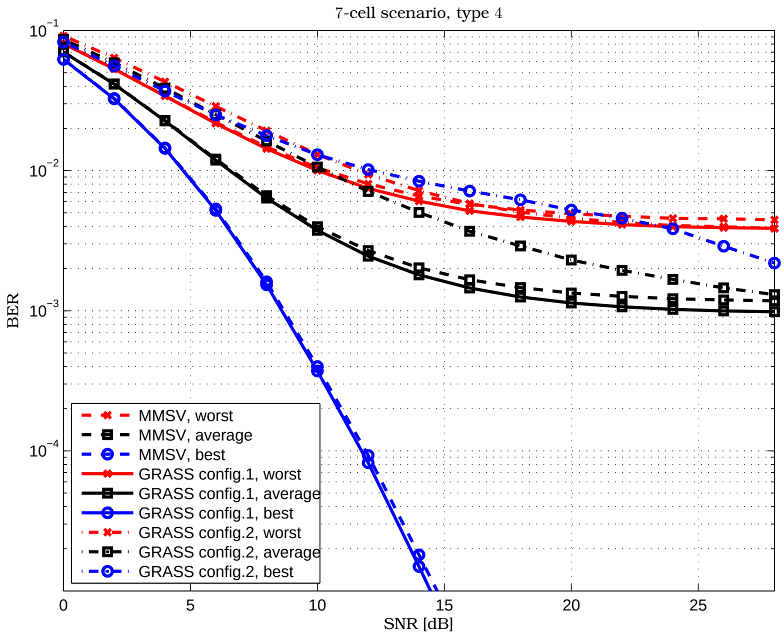

Figure 4.12: Bit Error Rate - $(3, 2) \times 3$ System.

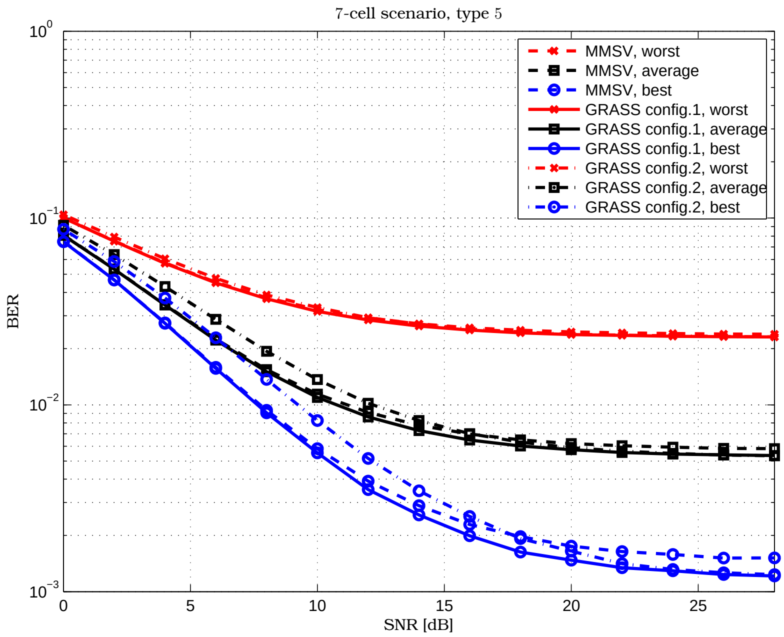

Figure 4.13: Bit Error Rate - $(3, 2) \times 3$ System.

4.3 Number of Iterations and NE Probability

As for GRASS with configuration 1, Figs. 4.14 and 4.15 show the NE probability (i.e., the probability of reaching a NE point), and the average number of game iterations, respectively. The NE probability decreases as the mutual MUI increases and becomes dominant compared to the noise factor in the denominator of Eq. (3.3). Consequently, the number of game iterations increases since the lack of NE implies the use of the fall-back algorithm MMSV triggered after λ iterations.

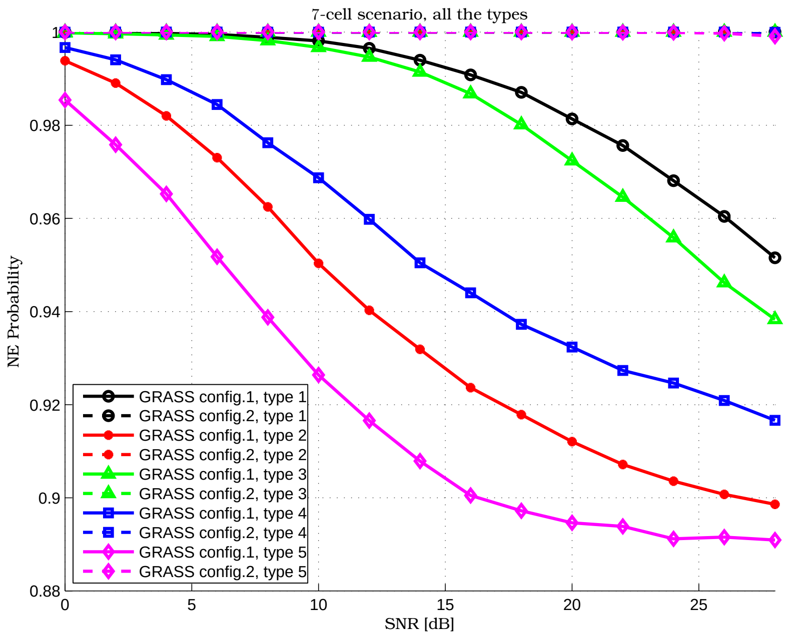

Figure 4.14: Nash Equilibrium Probability - $(3, 2) \times 3$ System.

With respect to GRASS with configuration 2, the behavior is completely different. The estimation error acts as a perturbation in the system (more precisely in the covariance matrix $\mathbf{R}_{-q}$). The NE probability does not decrease as the mutual MUI increases. It equals

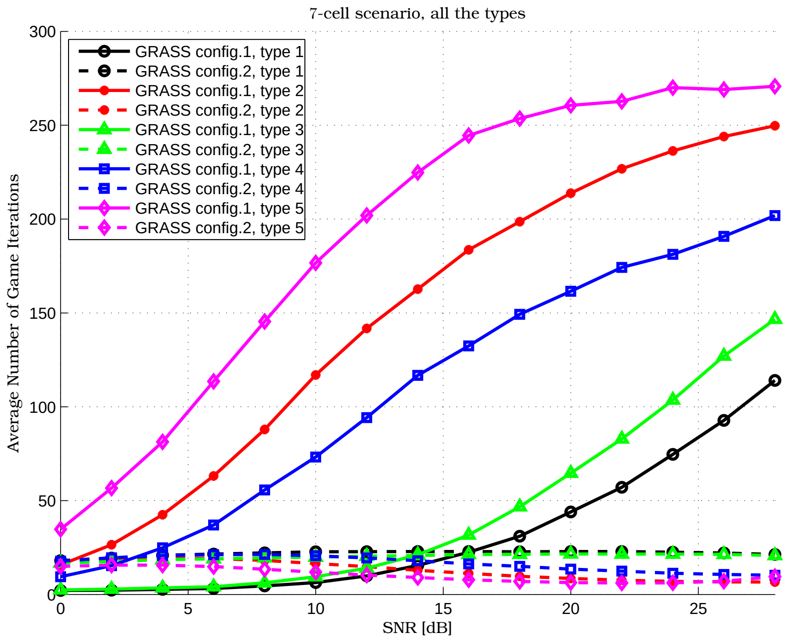

Figure 4.15: Average number of game iterations - $(3, 2) \times 3$ System.

approximately the unit as can be seen in Fig. 4.14, which implies that there is always a NE point for each channel realization. However, as GRASS with configuration 2 does not outperform GRASS with configuration 1 in terms of BER, these NE points reached are not good enough to improve the system performance. The number of iterations remains approximately constant at the value of 20 as the noise power decreases for scenario types 1 and 3. For the other scenario types, there is a slight decrease from the value of 20 to around 10.

4.4 Player Removal Analysis

In this analysis, only GRASS with configuration 1 (i.e., with a BSC) is considered. Only one BS is removed from the game at a time and the resultant performance (in terms of average BER) is compared to the performance of the conventional GRASS as well as the performance of the MMSV algorithm, both with all the players.

In Figs. 4.16 and 4.18, there is a significant performance loss by removing BS_2 in scenario type 1 and BS_1 in scenario type 3. This behavior occurs because those BSs observe low SIR levels, whereas the other BSs do not. Consequently, by removing those BSs the conflict of interests among the players decreases. The magnitude of that loss is such that the MMSV algorithm outperforms the GRASS approach. One reason is because the removed player remains to cause interference in the system and GRASS does not take into account such a source of interference anymore in order to find an equilibrium point. Clearly, the removal of another BS in those scenario types does not change the performance.

In Fig. 4.17, there is no performance loss when the BS_1 is removed from the game in scenario type 2. This occurs because that BS observes a high SIR, whereas the other BSs observe low SIR. On the other hand, if some other BS is removed from the game, the performance is degraded so that it equals the performance of the MMSV algorithm.

In Fig. 4.19, there is no performance loss when either BS_2, BS_3 or BS_4 is removed from the game. Those BSs observe high SIR compared to the other BS. However, a performance loss emerges when either BS_1 or BS_7 is removed from the game because those BSs observe the highest interference levels. Thus, the GRASS algorithm does not mitigate the interference received by those BSs anymore, decreasing the average BER. Removing some other BS, the

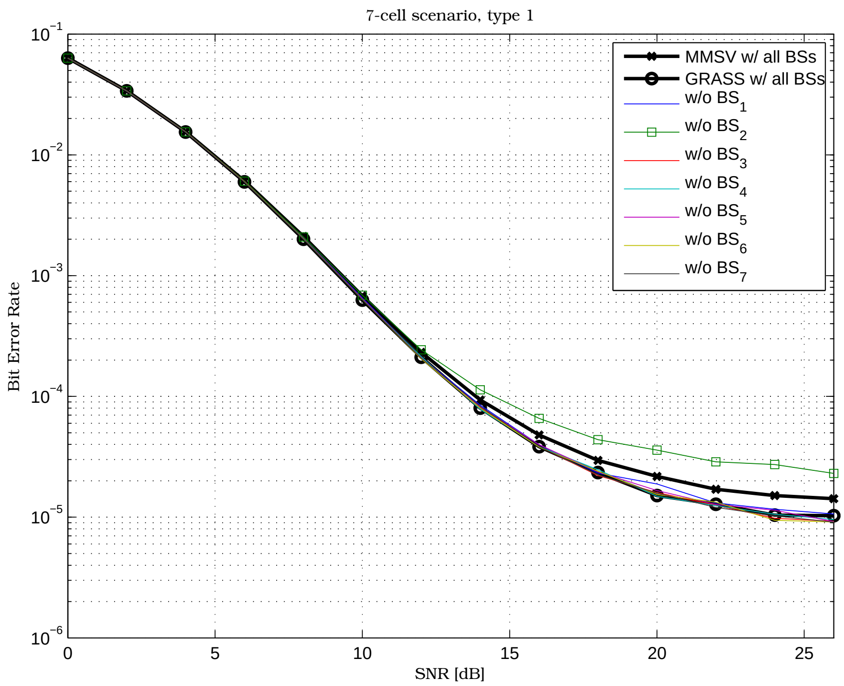

Figure 4.16: Player Removal Procedure - $(3, 2) \times 3$.

average BER does not change significantly and the BER curve lies between the MMSV and the conventional GRASS curves. Similar behavior can be seen in Fig. 4.20. Removing either BS_1 or BS_3, the performance loss is significant so that MMSV algorithm outperforms such an approach. On the other hand, removing some other BS, the average BER does not change significantly and the BER curve lies between the MMSV and the conventional GRASS curves.

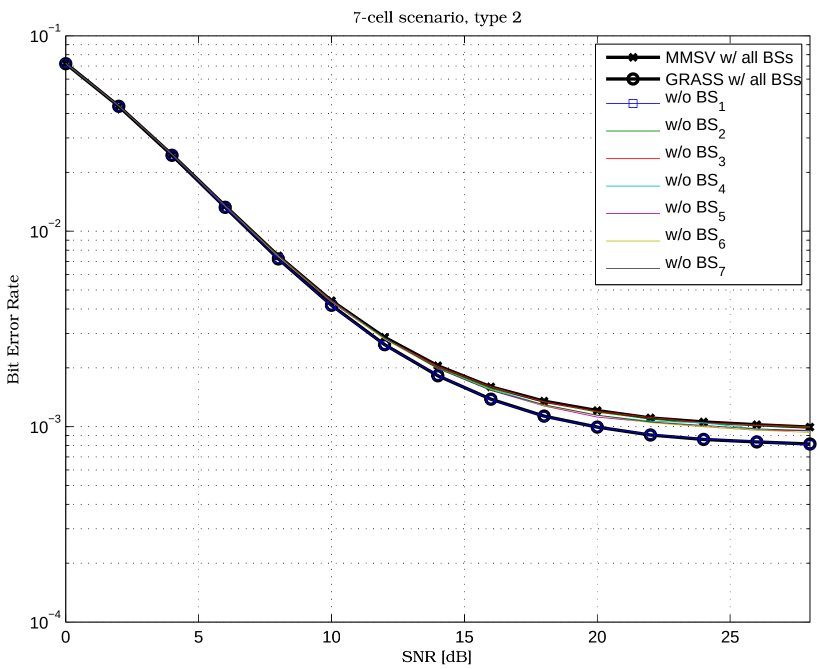

Figure 4.17: Player Removal Procedure - $(3, 2) \times 3$.

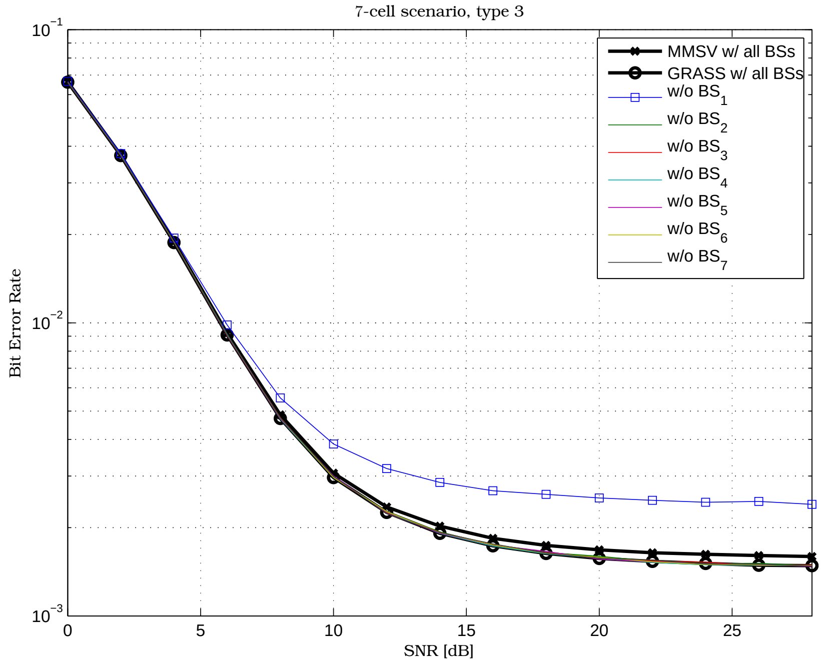

Figure 4.18: Player Removal Procedure - $(3, 2) \times 3$.

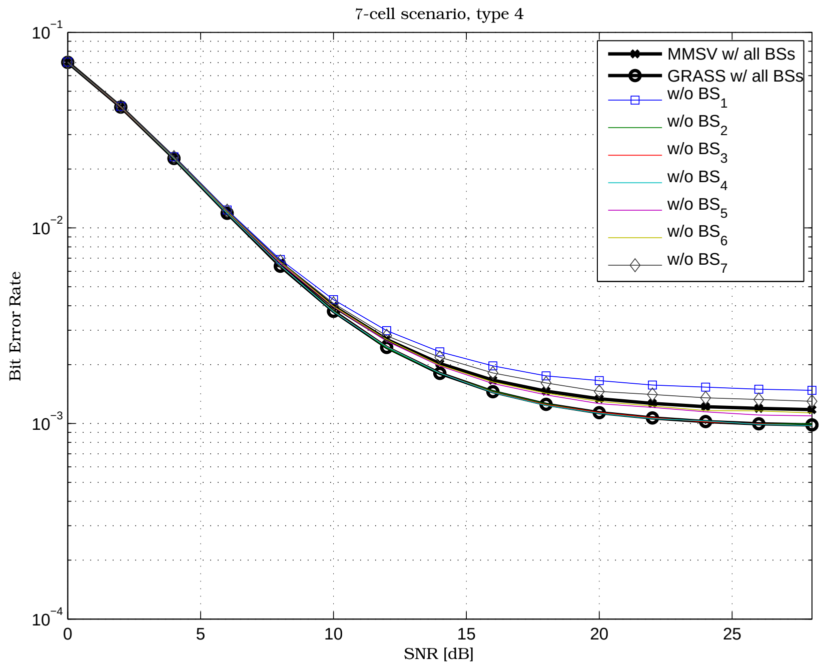

Figure 4.19: Player Removal Procedure - $(3, 2) \times 3$.

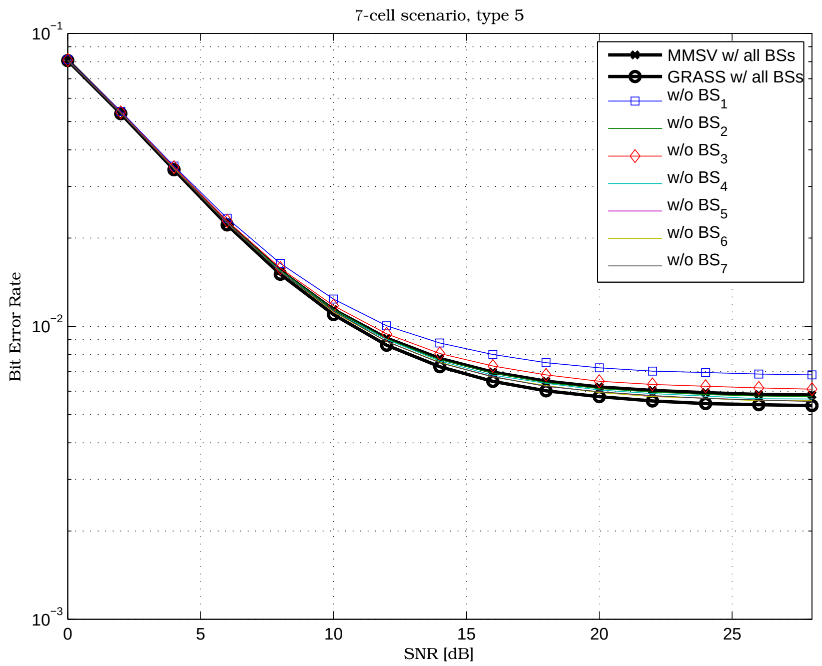

Figure 4.20: Player Removal Procedure - $(3, 2) \times 3$.

Chapter 5

Conclusions and Perspectives

Spatial multiplexing schemes are widely used in modern communication systems since they provide capacity gain through the employment of multiple antennas. In such systems, the cost of using several RF chains is high. Therefore, antenna subset selection algorithms emerge as a feasible way to decrease this cost.

Furthermore, a criterion for antenna selection was proposed based on a game theoretical approach that considers a competitive multi-user environment. The proposed algorithm, called GRASS, maximizes the minimum per-stream SINR and Monte Carlo simulations show that there is a significant performance gain compared to the criteria that do not consider MUI in terms of BER. Such gain increases when more receiver antennas are added on the BS side. It also increases when MUI increases to a certain level in which the interference can be mitigated by the MMSE receiver filter. However, if no BSC is considered in the system, estimation errors may degrade the estimate of the inferering channels. Such errors are large at a BS when the INR observed by that BS is low.

The GRASS algorithm can therefore be used to improve the performance of MIMO schemes providing a number of significant performance and implementation advantages, for many real-world operating scenarios. These advantages include:

- the amount of information exchanged among BSs is decreased due to the decentralized approach;
- the MUI is mitigated since the payoff function of the game takes into account the SINR;
- and the upper-bound λ is smaller than the number of interactions required by the exhaustive search algorithm.

Future work includes the analysis of the system performance with the use of new codebook designs, for the purpose of ensuring the existence of pure-strategy NE, and extension of our criterion to hybrid MIMO transceivers which take advantage of the fundamental tradeoff between diversity and multiplexing gains [32, 33]. In addition to those, the player removal procedure seems to be a good alternative to avoid BSs playing the game with degraded estimates of the interfering channels in GRASS with configuration 2. Also, the optimization of the parameter λ may avoid unnecessary search for an equilibrium point when such a point does not exist. Finally, the player removal procedure also leads to the possibility of choice of another kind of equilibrium concept, such as Stakelberg equilibrium [12].

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